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A REVIEW OF HYPERSONIC WAKE STUDIES

P. S. Lykoudis

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The RAND Corporation
SANTA MONICA • CALIFORNIA

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PREFACE

This Memorandum is a critical review of the fluid mechanics of hypersonic wakes formed behind vehicles of different geometries. Much progress has been made in the understanding of wakes during the last five years, but it appears now that further theoretical progress can only come after some detailed and rather difficult experimentation. The present survey, completed in January 1965, attempts to summarize and evaluate development in wake studies and also to outline the areas where future research should be undertaken.

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SYMBOLS

A = cross-sectional area

a = speed of sound

C_D = drag coefficient

C_p = specific heat at constant pressure

D = diffusion coefficient

d = body diameter

f = ratio of typical velocity at the axis of the wake divided by the free stream velocity

g = gravitational constant

H = altitude

h = enthalpy

j = indicates the dimensions of problem; has the value zero for a two-dimensional geometry and one for an axisymmetric geometry

k = thermal conductivity or a constant

L = length of ionization trail

M = Mach number

m = numerical constant

p = static pressure

$R(x)$ = function defining the shape of the shock

Re = Reynolds number

R_T = universal wake Reynolds number equal to about 14

R_w = wake Reynolds number based on velocity deficiency between turbulent front and axis and a length equal to the width of the turbulent core

r = radial coordinate

S = streamwise coordinate along the surface of a body or along the free shear layer

S^* = nondimensional coordinate measured from the point of separation along the shear layer; see Fig. 7

T = temperature

U = typical velocity in the wake along the direction of the mean flow

u = velocity along the direction of the mean flow

W = weight of reentry body

x = coordinate in the direction of the mean stream

GREEK

β = pressure gradient parameter as defined in laminar boundary-layer theory or wake turning angle

γ = ratio of specific heats

γ_L = fictitious ratio of specific heats used in Eq. (7)

δ = width of the wake in the radial direction or boundary-layer thickness

ϵ = eddy-diffusivity in incompressible flow

$\mathcal{E}$ = eddy-diffusivity in compressible flow

η = similarity parameter

Θ = momentum boundary-layer thickness

λ = characteristic length measured upstream from a point of separation; also nondimensional ratio defined in Fig. 13

μ = viscosity

ξ = nondimensional coordinate measured from the point of separation and along the shear layer

ρ = mass density

Ψ = portion of a fluid element that does not undergo distortions by shear forces during its flight

SUBSCRIPTS

c = value computed at the axis of the wake

D = value computed at the dividing streamline

e = value computed outside the diffusing core

f = value computed at the turbulent front

TR = value computed at the point of transition to turbulence

w = value computed at the wall

∞ = value computed at the free stream

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I. INTRODUCTION

But help me beat him off with all speed, and make full your currents
With water from your springs, and rouse up all of your torrents
And make a big wave rear up and wake the heavy confusion
And sound of timbers and stones, so we can stop this savage man
Who is now in his strength and rages in fury like the immortals.†

Since the time of Homer, wakes have always been associated with the confusion they leave behind them. In the lines above, Skamandros, the River of Troy, pleads with his brother Simoeis to team up with him and raise a wave, so that they can stop "this savage man" (no other than Achilleus) behind its wake. It appears, then, that the idea to use wakes for purposes of confusion and destruction is not novel with our age.

Five years ago there were probably less than ten papers in the literature on this conception of the wake, and today this review paper gives reference to more than a hundred and seventy-five. The motivation for so much work comes from Defense Department needs for understanding reentry phenomena and the interaction of hypervelocity vehicles with the atmosphere as a part of the ballistic-missile research program.

The first result that gives encouragement in understanding wake phenomena comes from straightforward kinematic application. If W is the weight of the reentering object, C_D its overall drag coefficient, A its cross-sectional area, ρ the variable mass density of the atmosphere, U the velocity, and H the corresponding altitude, then the equation of motion for a vertical reentry is

$$\frac{W}{g} U \frac{dU}{dH} = -C_D\left(\frac{1}{2} \rho U^2\right) A + W. \qquad (1)$$

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†Homer Iliad XXI. 311-315, translated by Richmond Lattimore, University of Chicago Press, Chicago, Ill., 1962.

Inspection of this equation reveals that for constant C_D and a given atmosphere the solutions are described by the dimensional parameter W/C_DA, a quantity known as the "ballistic coefficient." Detailed solutions of the above equation including different angles of reentry are available, for example, in Gazley[1] and Morris and Benson.[2] Let us assume that one can perform sufficiently accurate observations so that W/C_DA can be found by matching the observational data with the theoretical ballistic curves. However, more data are needed for the determination of the product C_DA if the weight is to be computed.

The next direction to examine is the distortion of the ambient atmosphere surrounding the object. Intuitively, it would appear that this distortion would be dependent on the size of the cross-sectional area of the object and its drag. This distortion can then be associated with some physical quantity whose change could be observed directly or indirectly. As such, the most primitive quantities are momentum and energy or, more directly, the velocity deficiency with respect to the free flight velocity and temperature above the ambient. Now the fluid comes to rest and the temperature reaches a maximum around the forward stagnation point; the question is how many body characteristic lengths it will take for the velocity deficiency and the temperature to assume values below a given measurable level.

To answer this question, let us for a moment concentrate on the mechanism by which cooling can occur. First, we have the mechanism of expansion. For hypersonic flight of blunt bodies (for which the distortions are most acute), the pressure decays downstream proportionally to the square of the Mach number and inversely proportional to the downstream distance x. The pressure is nearly ambient at a distance given by the relation[3]

$$\frac{x_{p \to p_\infty}}{d} \simeq \frac{M^2}{9}. \tag{2}$$

This length is rather short, relatively speaking. Even for a Mach number of 30 this distance is only one hundred diameters behind the body, whereas for M = 10 it is about ten diameters.

At the station $p \to p_\infty$, even allowing for cooling through the boundary layer over the vehicle, the temperature is still much above the ambient,

and further cooling will occur through a diffusion mechanism that could be laminar and/or turbulent. Let the temperature difference between the station $x_{p \to p_\infty}$ and the station x = L, where the temperature has decayed to the minimum value that an observer can measure above ambient be equal to $(\Delta T)_L$; if the diffusion coefficient, eddy-diffusivity, or thermal conductivity is denoted in general with D, and the width of the wake in the radial direction is denoted by δ, then for a characteristic difference of temperature across δ equal to $(\Delta T)_\delta$ the energy equation gives

$$\rho U C_p \frac{(\Delta T)_L}{L} \sim D \frac{(\Delta T)_\delta}{\delta^2}. \tag{3}$$

On the other hand, the order of magnitude of drag compu… with quantities evaluated behind the bow shock is proportional to the mass density ρ, the square of the velocity, and the square of δ. Using the definition of the drag coefficient C_D based on the dynamic pressure computed with free stream conditions and solving for δ, we may substitute in Eq. (3) from which we finally get

$$\frac{L}{C_D A} \sim \frac{C_p}{D} \frac{\rho_\infty U_\infty}{f} \frac{(\Delta T)_L}{(\Delta T)_\delta}. \tag{4}$$

In the above C_p is the specific heat at constant pressure, f is the ratio of the typical velocity in the wake divided by the free stream velocity, and A is the cross-sectional area of the object. We see immediately that the product $C_D A$ depends critically on how well we can measure the length of the trail L (based on the temperature $(\Delta T)_L$) and how confident we feel about our knowledge of the diffusion coefficient D and the temperature distribution in the trail. If we could establish this knowledge indisputably, then we could use Eq. (4) to calculate the product $C_D A$ and hence obtain the weight from the ballistic coefficient.

In one form or another, all the work undertaken to date attempts to provide this confidence. We can immediately see the difficulties ahead, because these attempts imply the solution of problems related to turbulence and separated regions, to say nothing of the chemistry that dictates the concentration and degree of excitation of the different species existing

in the air. Knowledge of these species is needed for the evaluation of information recorded through radar, optical infrared radiation, and other measurements. To be sure, the wake problem does not contain the statement of any new problems from the point of view of fluid mechanics but only focuses attention on a number of problems that, because of their difficulty, have been relatively neglected by the fluid mechanicians over the years.

Figure 1 shows schematically the different regions of distortion around a blunt or slender reentry vehicle. First, we recognize the strong forward shock wave, behind which we find an inviscid region. In the vicinity of the body, there is a boundary layer that will separate in the neighborhood of the corner C and develop downstream as a free shear layer. The products of the boundary layer form the "neck," in the neighborhood of which a recompression is formed through the "trailing shock." Between the neck and the base there is a recirculation region. Downstream from the neck we find a "diffusing inner core," whose behavior will be dictated by either laminar or turbulent transport properties, or both if transition occurs in this region. Outside this core, the fluid will carry with it the vorticity associated with the entropy generation created by the passage of the fluid particles through the forward, and to a smaller extent, the trailing shock waves.

For purposes of proceeding with our review in an orderly way, we focus our attention on the following regimes: laminar, transitional to turbulent, and turbulent; in a separate section we shall also discuss the chemistry involved. In the laminar regime we shall distinguish the neighborhood of the corner where the free shear layer starts, the recirculation region including the base and the neck, and the region downstream from the neck, up to the point of transition (if it occurs). Transition to turbulence will be reviewed only to the extent that it might occur in the wake itself rather than on the surface of the vehicle. This will also be true for the regions in which turbulence is present.

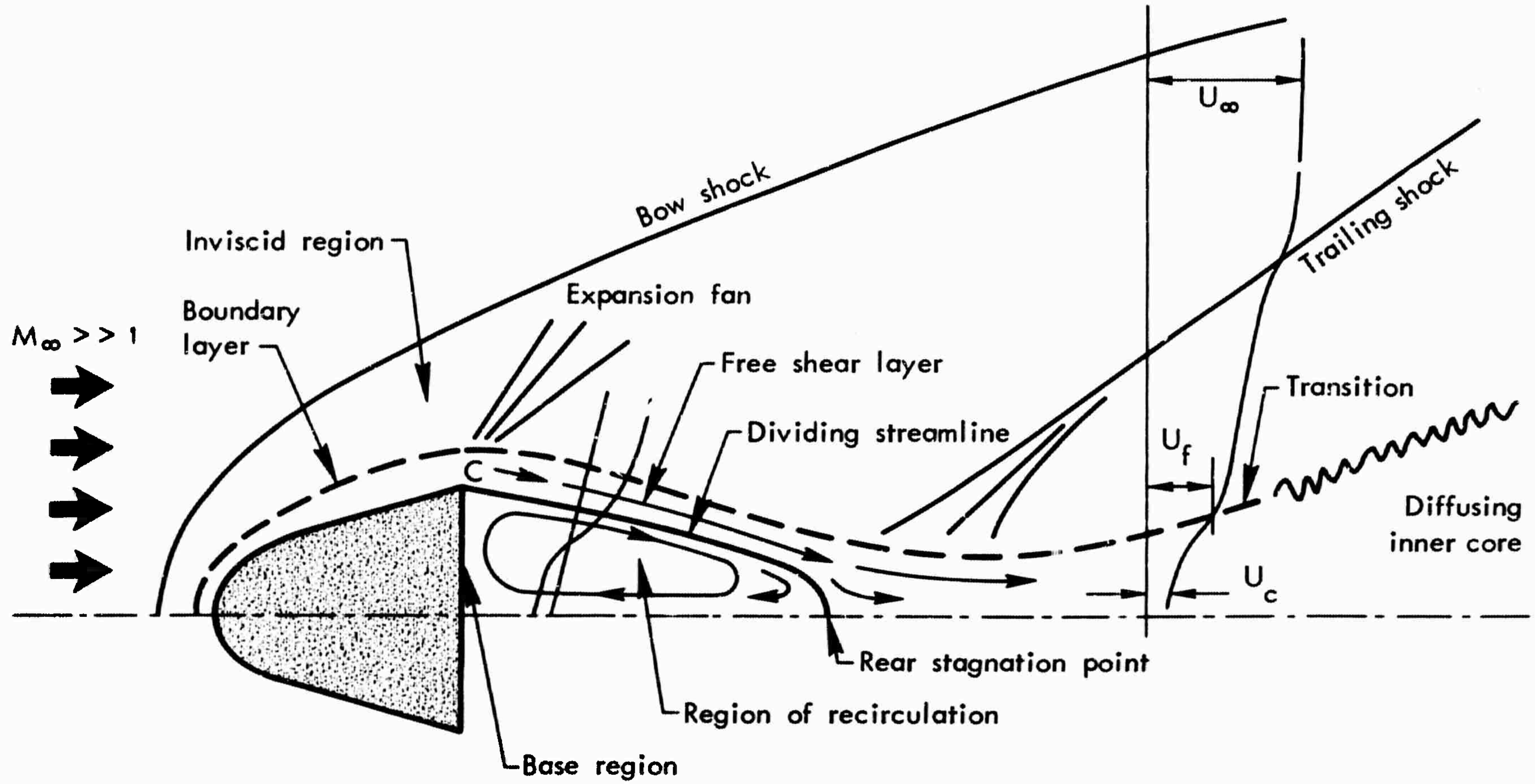

Fig. 1—Schematic representation of the different flow regions formed behind a body moving at hypersonic velocity

II. THE LAMINAR REGIME

A. The Expansion-conduction Models

Perhaps the first attempt to describe the wake behind a blunt body from a fluid mechanical point of view is due to Feldman.[4] He considered a hemisphere-cylinder configuration in which the velocity deficiency and the temperature fields were caused by a strong, almost parabolic, bow shock wave. The flow field was separated into two large regimes: (1) a region extending a sufficient number of body diameters aft to a point where the free stream pressure was recovered, and (2) the region beyond that. The influence of the boundary layer, the base, the recirculation region, the neck, and the diffusing core were not considered in this first attempt. The first region was treated numerically by the method of characteristics, which allowed for an isentropic expansion of every streamline, starting just behind the bow shock wave. Because different streamlines are associated with different irreversibilities, the greatest irreversibility occurs at the normal part of the shock, and deficiencies of velocity and surpluses of temperature persist at the end of this region even though the pressure becomes ambient. To compute the temperature and other profiles, from this point on, an integral calculation was made, based on a cooling mechanism due entirely to thermal conduction in laminar flow. Although real gas properties were used, the flow was assumed to be in thermodynamic equilibrium.

The involvement of air chemistry rates in such calculations was outlined numerically at that time by Lin,[5] whereas calculations similar to those of Feldman, with and without consideration of chemistry, were undertaken by Goulard and Goulard.[6] Similar calculations were also made by Ting and Libby[7] and Lew and Langelo.[8] Lykoudis[3] considered the same problem in an approximate way and obtained solutions in closed form; the results compared well with the numerical computations of Feldman.

We now know that the results of all these papers are not consisten with reality, because at the lower altitudes, where the flow has a tendency to be in thermodynamic equilibrium, the "diffusing core" will be turbulent, whereas at the higher altitudes, where the flow might be

laminar, the flow will not be in thermodynamic equilibrium. However, the above model is still a useful one in the sense that it is believed to ascribe the conditions prevailing outside the "turbulent front." What follows is a brief summary of these first results taken from Ref. 3.

For a blunt body, the bow shock wave is, within a good approximation, a parabola, and if $R(x)$ denotes its shape with respect to the distance x measured downstream from the point where the shock is normal, then the strength of the shock at any station x is given by the slope dR/dx. Given a streamline entering the bow shock wave with ordinate R, one can use the oblique shock wave relations corresponding to dR/dx to find the flow conditions behind the shock and then expand isentropically for every streamline, separately, until the ambient pressure is reached. For each streamline, equivalent adiabatic exponents can be used to take into account real gas effects. From a conservation of mass argument one deduces that

$$\rho_\infty U_\infty R \, dR = \rho U r \, dr, \tag{5}$$

where r is the radial distance from the axis at which a streamline entering the bow shock wave finds itself at the station where $p \to p_\infty$. U_∞ and U are the free stream velocity and the velocity at the station where the pressure is ambient. Because $f = U/U_\infty$ is about 0.8 (as will be shown later), it can be seen that R is approximately equal to the familiar compressible Howarth variable. The shape of the bow shock is given from blast-wave theory approximately by

$$R = k\sqrt{x}, \tag{6}$$

where k is a constant. For the enthalpy distribution, the above considerations give

$$\frac{h_{p \to p_\infty}(R)}{h_{p \to p_\infty}(0)} = \frac{1}{\left[1 + \frac{4}{k^4} R^2\right]^{1/\gamma_L}}, \tag{7}$$

where γ_L is an equivalent specific heat ratio equal to about 1.2. In

the neighborhood of the centerline, Eq. (7) behaves like a parabola and its form is in general Gaussian with a Gaussian depth equal to $k^4\gamma_L/4$. As a matter of fact, if the Howarth variable is introduced in Eq. (7) so that the factor f enters into the depth, this depth is practically equal to C_D, since from blast-wave theory k^4 varies as the inviscid drag coefficient C_D. Feldman,[4] guided by his numerical results, had selected the value 1 for this depth; since for a blunt body very nearly $C_D \simeq 1.0$, the Feldman value was thus identified with the drag coefficient.

Furthermore, assuming that the stagnation enthalpy is conserved, one can show[9] that the factor f is given from the relation

$$f \approx (1 - M_\infty^{-1/3})^{1/2}. \tag{8}$$

Also the enthalpy at the station $p \to p_\infty$ is equal to

$$\frac{h_{p\to p_\infty}(0)}{h_\infty} \approx (1 - f^2)(\gamma - 1)\frac{M_\infty^2}{2} \approx \frac{M_\infty^2}{15}. \tag{9}$$

From Eq. (8) it can be seen that for Mach numbers between 10 and 30, f is about equal to 0.80. This result is verified through numerical characteristic calculations.[4] Note that the stagnation enthalpy $h_s/h_\infty = 1 + [(\gamma_\infty - 1)/2]M_\infty^2 \approx M_\infty^2/5$ is only three times as high as the enthalpy given by Eq. (9). As a matter of further interest, and again within the above approximation, the local Mach number where $p \to p_\infty$ can be computed by the relationships

$$M_{p\to p_\infty} \approx f\frac{U_\infty}{a_{p\to p_\infty}} \simeq \frac{f}{\sqrt{\left(\frac{\gamma - 1}{2}\right)(1 - f^2)}}. \tag{10}$$

For $f \simeq 0.8$ and $\gamma = 1.4$, the above yields a Mach number equal to about 3, which is independent of the free stream Mach number. This is, of course, only true for blunt bodies in thermodynamic equilibrium. Nonequilibrium effects lower the value of γ, and hence the Mach number in question will increase (as an example, for $\gamma = 1.2$, $M_{p\to p_\infty} = 4.2$). For slender bodies, the Mach number values at the station where $p \to p_\infty$ are higher and depend on the free stream Mach number, which is approached as the inviscid drag

becomes smaller and smaller. This relation will be invoked when we shall discuss the conditions under which transition to turbulence is possible.

Further downstream from the point $x_{p \to p_\infty}$, the enthalpy profile given by Eq. (7), or the equivalent information in numerical form, can be used to evaluate cooling by the mechanism of thermal conduction. The result, assuming an average thermal conductivity coefficient (computed through the practically constant Prandtl number), is the following:

$$\frac{h(0)}{h_{p \to p_\infty}(0)} \simeq \frac{1}{1 + \frac{16}{(Pr)k^4 \gamma_L} \frac{\bar{\mu}}{\mu} \frac{1}{Re} \frac{x}{d}} . \qquad (11)$$

Variable conductivity complicates the above expression slightly.[3] Knowing the enthalpy from Eq. (11) and the ambient pressure, any equilibrium property of the laminar wake can then be computed. The most important property is perhaps the electron concentration, which is critical in evaluating the interaction of an electromagnetic wave with an ionized trail. The order of magnitude of the electron concentration at the point $x_{p \to p_\infty}$, depending on velocity and altitude, could be as high as 10^{13} e/cm^3; and since the ionosphere in the background has a concentration of the order of 10^6 e/cm^3, a radar operating at a frequency corresponding to 10^9 e/cm^3 would identify a length for the trail corresponding to a decay from 10^{13} to 10^9 e/cm^3. Within the above assumptions one finds that for a blunt body 10 cm in radius, this radar trail for an altitude of, say, 60,000 ft and for a free stream velocity of 17,500 fps, would be about 5 km (long). As mentioned previously, turbulence mixes the "diffusing core" much faster and the trail should be expected to be much shorter; however, the reflection properties are different, and the return signal could be stronger for the same length. A few more remarks are appropriate with respect to the iar expansion-conduction solution. It has been found numerically by Feldman[4] and Lees and Hromas,[10] and analytically by Lykoud s,[3] that the influence of the trailing shock wave is very small with regard to the details of the inviscid flow in the far wake.

In Ref. 11 Lykoudis examines the validity of the assumption of

considering separately the regions of expansion and conduction. From an equation similar to Eq. (2), one can establish that the length it takes for expansion varies as M^2, whereas from the energy equation the length it takes for cooling through conduction varies as (Re), where Re is the Reynold number. The ratio of these two characteristic lengths thus varies as M^2/Re. The analysis has yielded that the centerline enthalpy may be calculated with sufficient accuracy using two separate regions if the above parameter satisfies the condition

$$M^2/Re \leq 25.$$

In the above, Re is the Reynolds number based on the sphere radius and free stream conditions. It is thus concluded that expansion and conduction are equally important at high altitudes (small ρ_∞) and for small bodies and low flight velocities. These are precisely the same conditions under which chemical relaxation effects become important.

During an actual reentry the free flight velocity changes continuously with altitude. We wish to know what bearing the steady-state solutions has on the nonsteady problem and more precisely how the length of the wake will vary during an actual reentry. Through an order of magnitude analysis one can establish that the time taken by a fluid particle to travel the characteristic length of the body is much smaller than the time it takes the vehicle to change its speed substantially.† As a result the wake will develop temporally at each fixed altitude as if the vehicle had moved past that point with a constant velocity equal to the one it had when passing through this point. The length based on this procedure has been called the "ballistic length." Another wake length can be computed if one assumes that at a given altitude the wake's length is equal to the one that eventually would develop if the body had remained at that altitude moving with a constant speed. These two lengths are not equal, and it is obvious that the

†See also the work of Zeiberg[12] for a study of such effects as vehicle deceleration and free stream density variation. Zeiberg[13-15] has also examined the behavior of the hypersonic wake behind an oscillating body.

second method of computation is erroneous. Klarimon[16] and Lykoudis[17] examined this question and found that the "ballistic length" remains practically constant with altitude, whereas the "constant altitude" and "velocity wake length" decrease very abruptly at the lower altitudes.

B. The Flow Upstream and Downstream from a Corner of Separation

Consider the flow in the neighborhood of the point C as shown in Fig. 1. We pose the question to find how this corner will influence the flow upstream and what the flow will be like in the free shear layer downstream. Goldstein[18] was perhaps the first who made an attempt to calculate the flow in the downstream direction from the trailing edge of a thin flat plate. He started out at the corner assuming a Blasius velocity profile and found that analytic expansions did not converge and that an answer could be obtained only by a step-by-step numerical procedure. The asymptotic wake behind this configuration, first investigated by Tollmien,[19] was found to have a velocity profile resembling a Gaussian distribution. When compared with Goldstein's results, this profile is found to be valid after a distance approximately three times as large as the length of the plate. On the other hand, from physical considerations one expects to find at the trailing edge a velocity profile with a zero derivative rather than the Blasius-like profile assumed by Goldstein. To this end, Goldburg[20] made an attempt to describe the flow upstream from the trailing edge of a flat plate. He made two lengthy expansions in the neighborhood of the trailing edge, one in terms of $(Re)^{-1}$ and the other in terms of $(Re)^{-\frac{1}{2}}$. He concluded that neither was successful in satisfying all the boundary conditions of the problem, and the matter is still unsettled; it is intuitive, however, that the characteristic length of influence upstream of the trailing edge will get smaller as the Reynolds numbers become higher.

We now come to the case of a free shear layer, which is of more practical interest than the flat plate. Chapman[21] was perhaps the first to examine a problem of this type. He started out with a two-dimensional corner with a base of infinite extent and he assumed that the velocity profile at the corner was of zero boundary-layer thickness, and that the free shear layer mixing would occur at constant pressure. He showed that

the problem reduced mathematically to the Blasius differential equation with different boundary conditions; namely, in terms of the similarity parameter η, the velocity component in the direction parallel to the free stream should be zero at minus infinity and equal to the free stream velocity at plus infinity. He solved the problem for a Prandtl number equal to one and with a viscosity law given by a power law in terms of the temperature. The main result of this analysis is the computation of the velocity at the "dividing streamline." This is the streamline that eventually must stagnate in the neighborhood of the neck. The flow above must continue on through the neck, intersect with the trailing shock, and eventually diffuse in some manner farther downstream. This streamline is shown in Fig. 1. From the assumption of zero original boundary-layer thickness, Chapman's solution emerges as an asymptotic one valid only when the Reynolds number is very high. Figure 2 shows the velocity profile in the customary similarity plane.

It was pointed out by Denison and Baum,[22] and elsewhere, that the length between the corner and the neck is hardly long enough to permit this assumption for the usual geometric configurations of interest. On the other hand, it should be pointed out that for an infinite Reynolds number, regardless of the initial profile chosen at the corner C, the value of the dividing streamline velocity should always approach the Chapman value, which is 0.587 times the free stream velocity.

The first solution of the Chapman problem with a boundary-layer thickness different from zero is due to Denison and Baum.[22] They started out at the corner with a Blasius profile, which was justified by the argument that for hot streams and cool walls the development of the boundary layer is not influenced very much by the pressure. Their solution, by necessity, is a nonsimilar one since similarity is destroyed by the finite thickness of the initial velocity profile. The calculated velocity profiles are given in terms of a nondimensional parameter S*, which in essence is the square of the ratio of the two momentum boundary-layer thicknesses--the local one at a station a distance S downstream from the corner, and the thickness over the body at the station S = 0. If this ratio is Θ/Θ_i we have

$$\left(\frac{\Theta}{\Theta_i}\right)^2 = \left(\frac{S}{\Theta_i}\right)^2\left(\frac{\Theta}{S}\right)^2 \approx \left(\frac{S}{\Theta_i}\right)^2 \left(k/\sqrt{(Re)_S}\right)^2 = k^2\left(\frac{S}{\Theta_i}\right)^2 (Re)_S^{-1} = k^2\xi , \qquad (12)$$

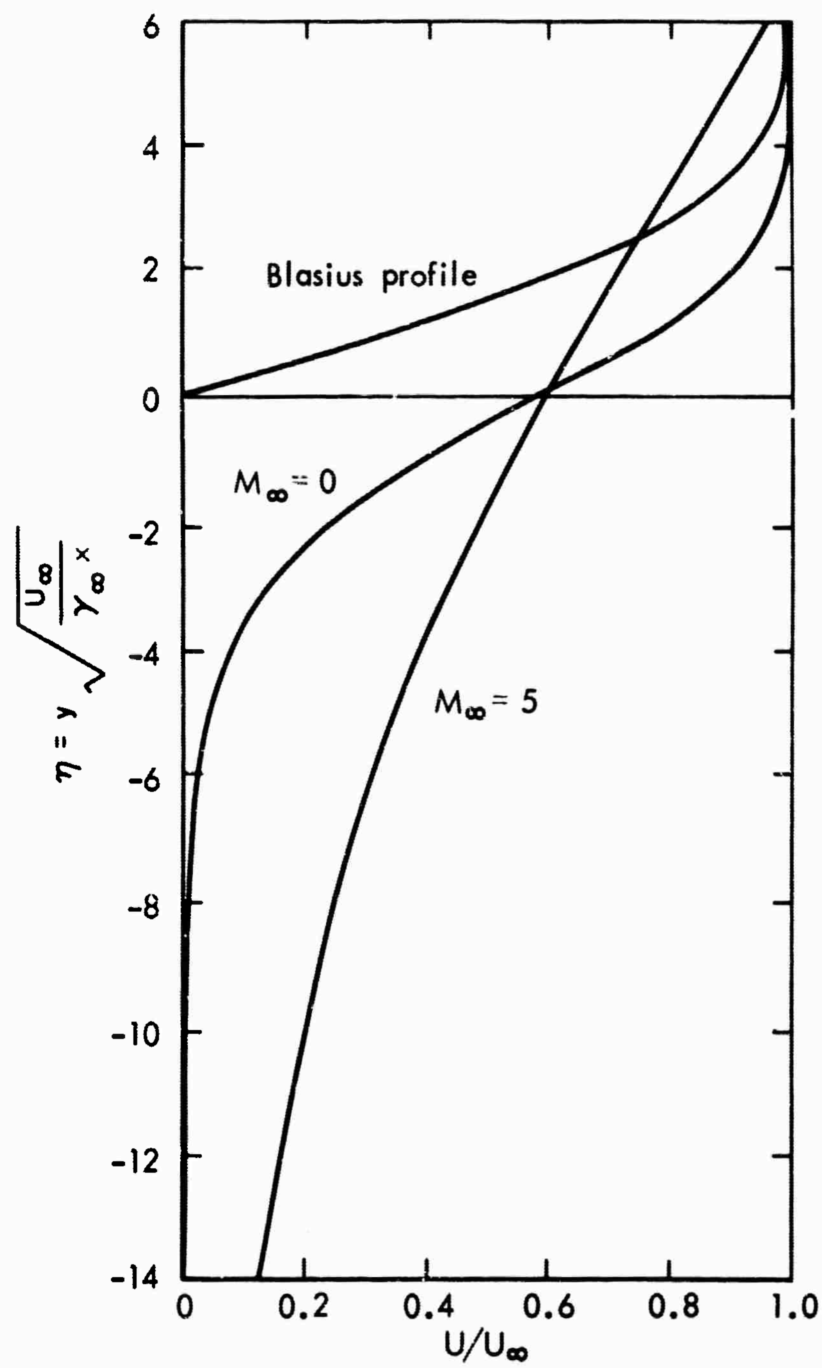

Fig. 2—Comparison of the Blasius profile and the velocity profiles in a constant pressure mixing layer according to Chapman[21]

where k is a constant. The quantity ξ is used by Kubota and Dewey[23] where the relation between S* and ξ is shown to be $S^* = f_w''^4 \xi$; f_w'' is the Blasius shear at the wall. For S* tending to infinity, one gets the Chapman solution, whereas for $S^* \to 0$ the Blasius profile is recovered at the corner of separation. The point where $S^* = 0$ is a singular point. An analytic expansion is used in this neighborhood, as suggested by Goldstein's[18] solution, in order to start the step-by-step numerical integration.† The value of the velocity at the dividing streamline varies between zero and the Chapman value for S* equal to zero and infinity, respectively.

Chapman et al.,[25-27] and independently Korst et al.,[28] have forwarded the idea that for the calculation of the pressure in the dead-air region one could assume that the compression along the dividing streamline and through the trailing shock takes place isentropically. However, when the numerical results of Denison and Baum[22] are used with the above scheme, it is found that the results in terms of the pressure or the turning angle are independent of the Reynolds number,‡ contrary to experimental evidence. In particular, the base pressure is higher than the one computed using the Chapman asymptotic value for the dividing streamline velocity. Comparison in Ref. 22 with available experiments verifies that the Chapman-Korst et al. scheme is a good one at the high Reynolds numbers, whereas at the lower values the experimental points agree more with the constant value derived through the Denison-Baum solution. Measured free shear layer turning angles show similar agreement when compared with angles computed with Chapman's asymptotic solution or with the Denison and Baum results. These comparisons are shown in Figs. 3 and 4 taken from Ref. 22.

All of the above numerical calculations are made by matching an inviscid with a viscous solution as follows: After selecting a wake angle at the corner, the flow is expanded isentropically and then

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†For a general discussion of this point, see Ref. 24, which also concerns similar blowing profiles as starting profiles.

‡This occurs because the parameter S* is the ratio of two lengths, both of which are proportional to the radius of the base. See insert, Fig. 7.

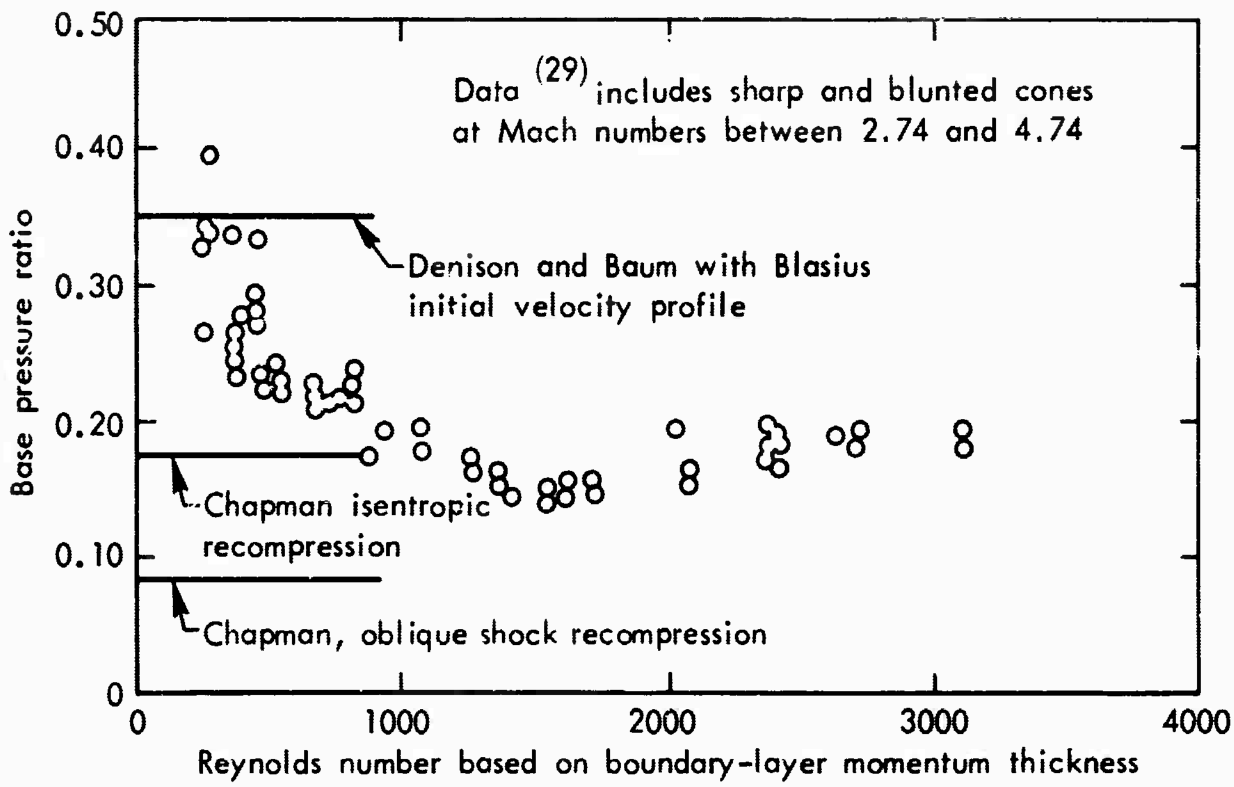

Fig. 3—Comparison between experiment and theory for base pressures at different Reynolds numbers

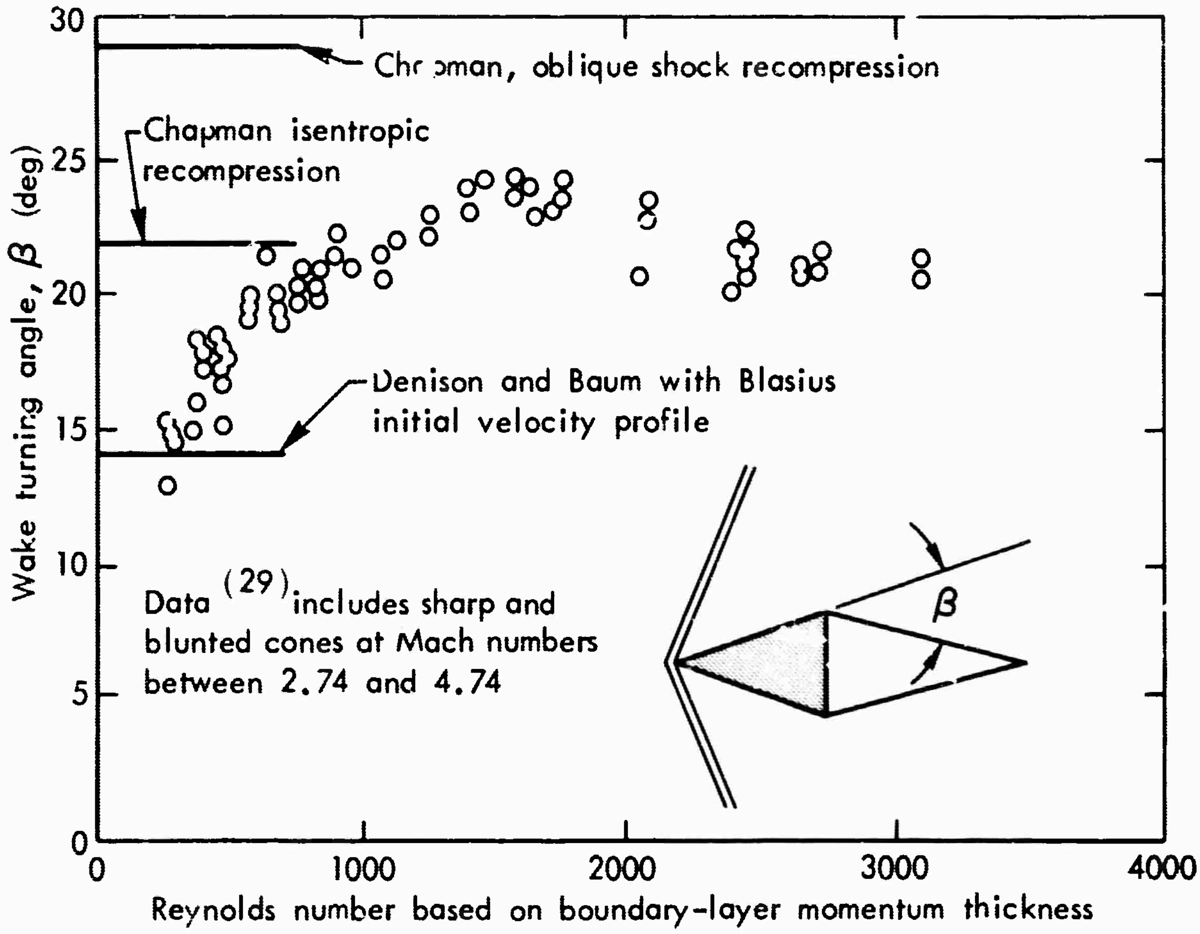

Fig.4—Comparison between experiment and theory for the wake turning angle for various Reynolds numbers

recompressed again, either isentropically or by an oblique trailing shock. It is then assumed that the wake angle is the correct one if the value of the pressure calculated as above (without viscous effects) is the same as the one derived by using the Chapman-Korst hypothesis in conjunction with the velocity on the dividing streamline reached at the neck.

The work of Denison and Baum(22) has stimulated further work at Electro-Optical Systems, Inc., by Baum and King (see Refs. 24, 30-35). In Ref. 36 computations are carried out for the evaluation of enthalpy and atom profiles under the assumption that all the thermal and mass diffusion coefficients are equal. The computations are made for zero heat transfer at the base. Here again as in Ref. 22 the pressure is assumed to be constant in the shear layer and the initial velocity profile is assumed to be of the Blasius form.

In Ref. 30 Baum solves the same problem treated by Denison and Baum(22) except that he starts with a modified Blasius profile to take into account mass transfer from the body surface. As expected, the base flow properties change significantly; first, the shear layer is increased in thickness as is also the base pressure; at the same time the dividing streamline velocity decreases, along with the core enthalpy in the recirculating region and the dividing streamline stagnation enthalpy. The turning angles are decreased.

In Ref. 35 King and Baum study the effects of base bleed on the laminar base flow. As a result of mass blowing at the base, one expects to find more drastic changes in the flow than from simple blowing from the surface of the vehicle. The assumptions made are similar to the ones made in the previous papers reviewed immediately above. Most important of the assumptions made is that the boundary conditions at the free shear layer are unaltered by the injection. Qualitatively, the conclusions are the ones that one would expect on physical grounds. For instance, it was found that for sufficiently high base blowing rates the near wake becomes longer (corresponding to smaller turning angles), so that in the limit of high blowing rates the shear layer approaches the Chapman limit. As the near wake becomes longer the wake also becomes cooler.

In Ref. 32 King continues the generalization of the central ideas of Denison and Baum[22] and performs heat transfer calculations at the base. Since all the previously reported work at Electro-Optical Systems was limited by the assumption of an adiabatic condition at the base, with a temperature in the recirculating region equal to the base temperature, this work was undertaken to remove this limitation. First, it was assumed that the recirculating flow region is determined by an incompressible potential flow. The potential flow is then coupled with the separated shear layer through a mass conservation argument--a condition that fixes the strength of the mass source. After solution of this inviscid problem, a boundary-layer solution follows in which the Cohen and Reshotko[37] boundary-layer analysis is used. It is found that the thermal resistance at the base is very high, so that even if the base wall temperature is absolute zero, the resulting heat transfer does not affect the wake. It is also found that the base pressure distribution is approximately parabolic and that the deviation from a uniform pressure at the base becomes more severe at low Reynolds numbers, a result which is in qualitative agreement with experiments by Kavanau.[38] The theory breaks down for small Reynolds numbers.

In Ref. 31 Baum undertakes a calculation in which he uses the assumption that a Blasius profile at the corner C turns through an isentropic expansion. The resulting velocity profile is used to compute the free shear layer, as it was done by Denison and Baum.[22] As an immediate result of this new model, the velocity at the dividing streamline becomes higher for a given degree of expansion and the base pressures are smaller; at the same time the base region becomes shorter. On the other hand, for a given wake length the core enthalpy increases somewhat as the sharpness of the turn increases. These effects are depicted in Figs. 5, 6, and 7.

In Ref. 33 King tabulates the properties of base flows for cones and wedges, computed on the basis of the work reported above at Electro-Optical Systems. The same author in Ref. 34 compares turning angles obtained from ballistic range and wind tunnel photographs of cones in free flight at low supersonic Mach numbers ($1.5 < M < 5$) with those obtained theoretically by the nonsimilar free shear layer theory and an isentropically expanded Blasius profile at the corner. The comparison (see Fig. 8) shows

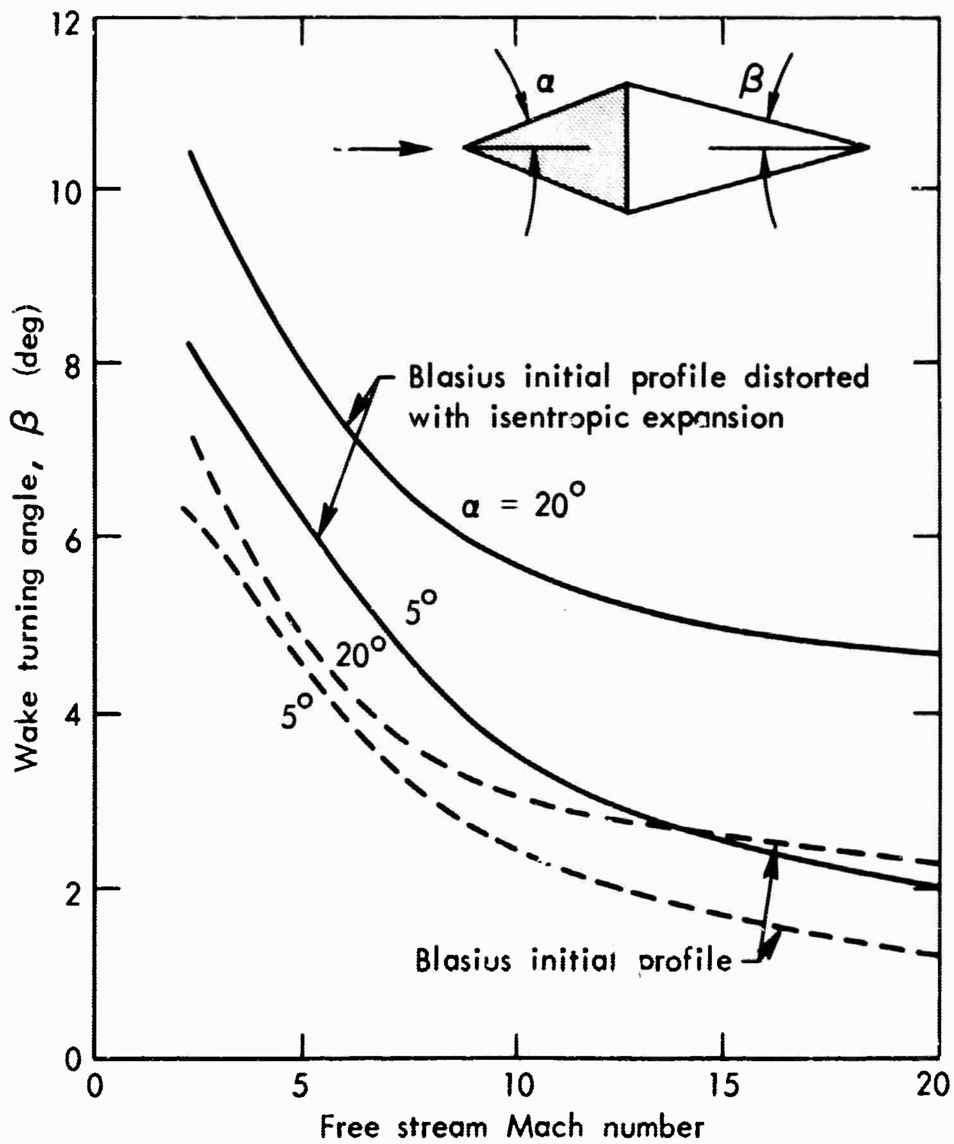

Fig. 5—The effect of a sharp turn at the shoulder of a cone on the wake turning angle

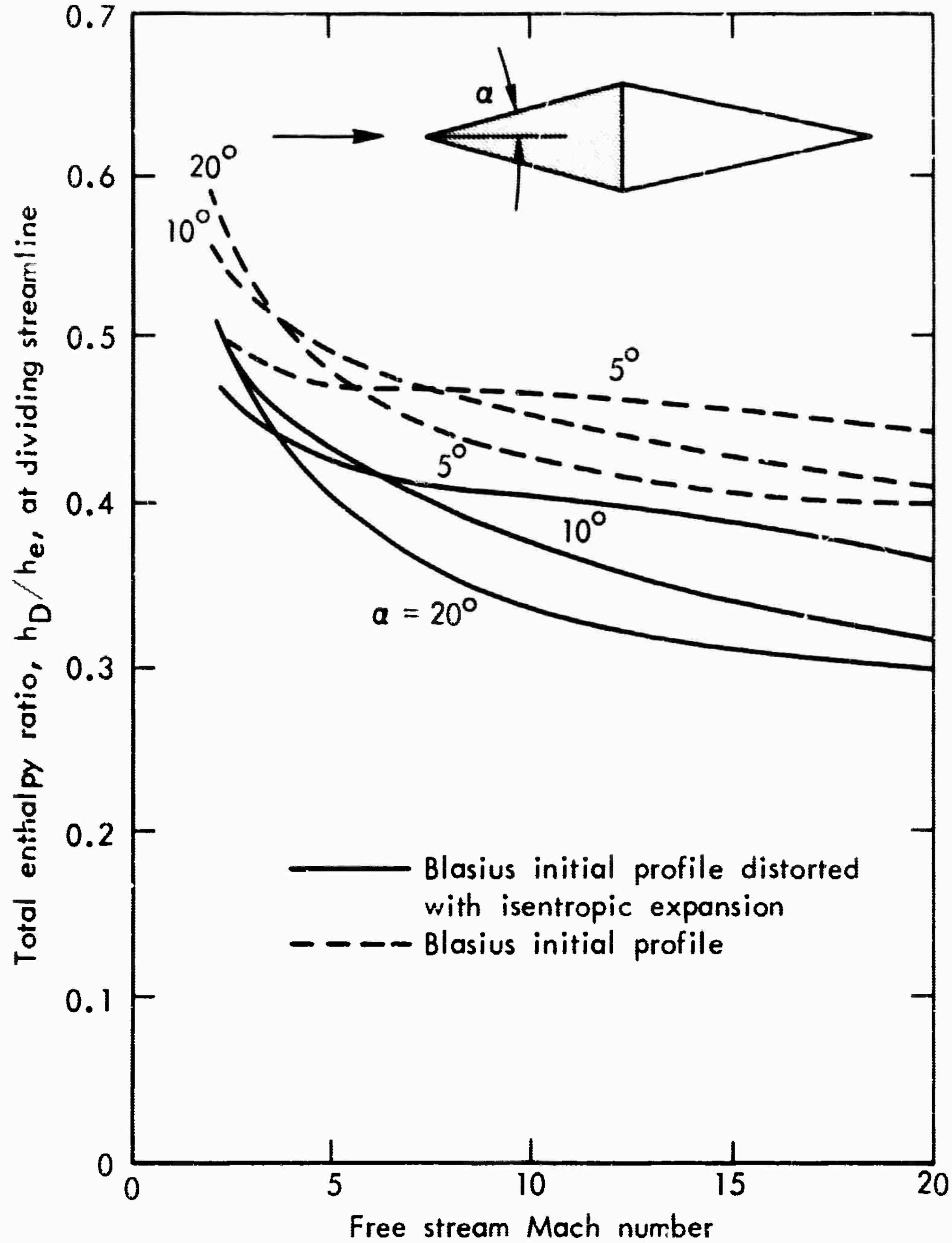

Fig. 6—The effect of a sharp corner at the shoulder of a cone on the dividing streamline total enthalpy

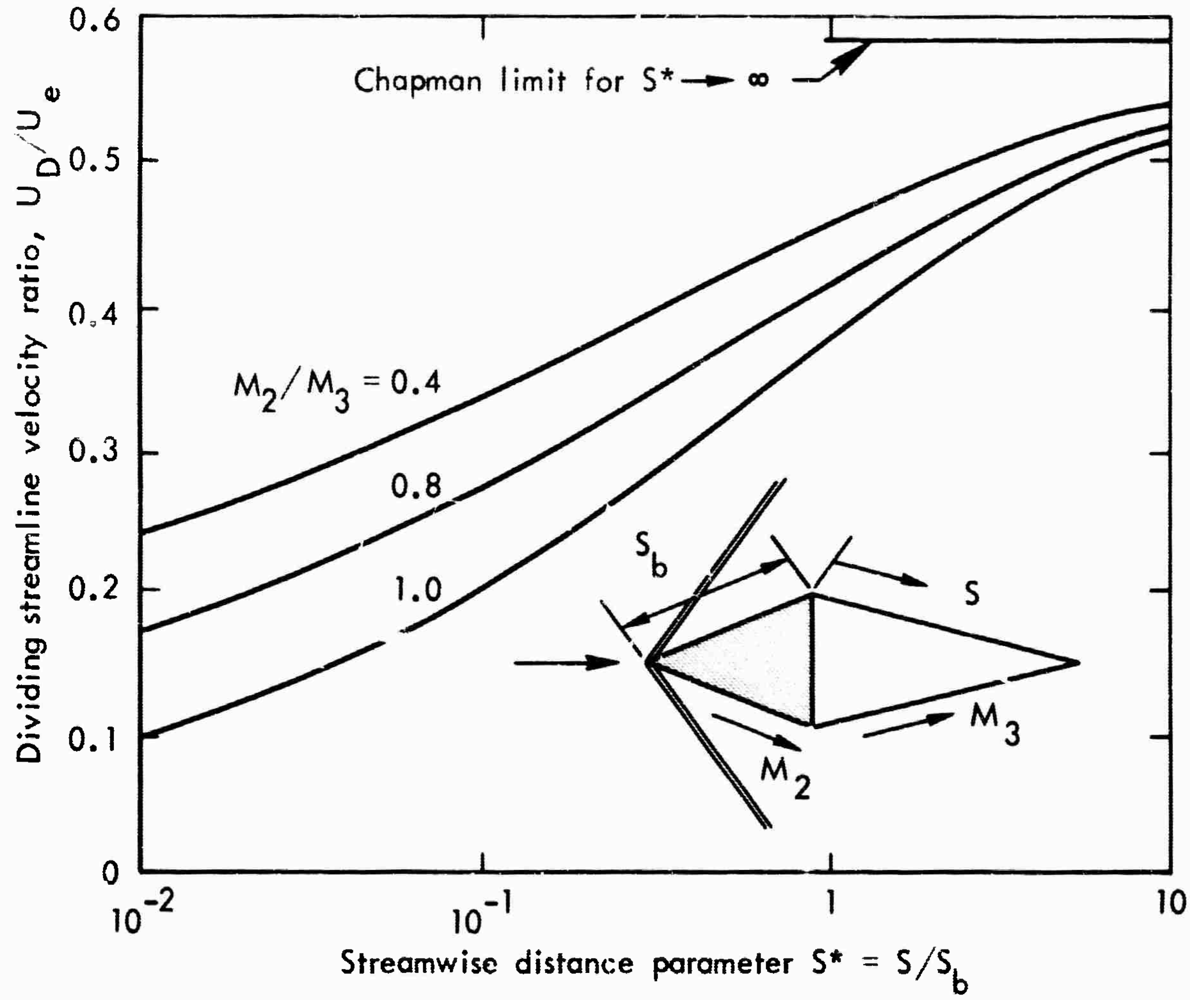

Fig. 7—The effect of an isentropic expansion of a Blasius profile on the dividing streamline starting at the body shoulder

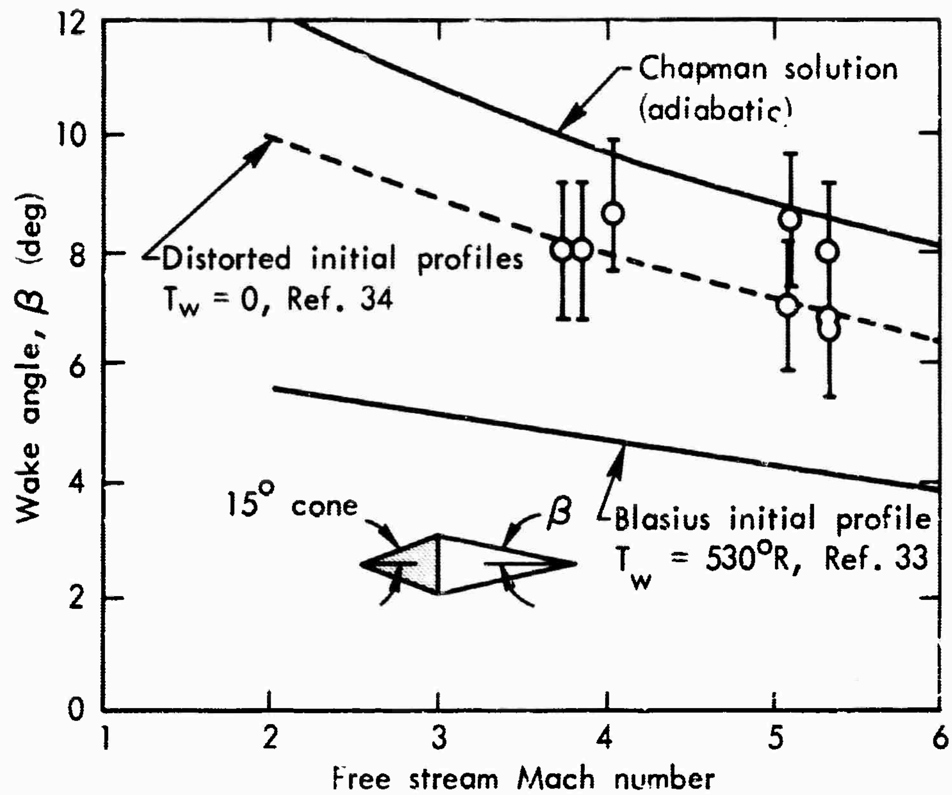

Fig. 8—Comparison of experimental wake turning angles with the theories of Chapman et al.[27] and King[34]

that the effect of the inclusion of the isentropic expansion is to bring the theory and the data much closer.

An abortive attempt to introduce the base scale length in the free shear layer calculations was undertaken by Lykoudis in Ref. 39, in which the Chapman free shear layer was reconsidered for the case in which the velocity vanishes at a distance of the order of the base radius rather than at infinity.

Slow flow solutions in the base region are under investigation at Avco-Everett and will be reported in the literature soon.[†] In the sense that these solutions are also worked out independent of the free shear layer and recompression region, they will be open to the same criticism as similar work. For completeness we should mention the work of Elrod[(40)] in which he extends the Chapman analysis of the laminar free shear mixing with results that coincide with those of Ref. 22 in the case of a Blasius initial velocity profile at the corner. Elrod's method is different, however, in the sense that he studies a class of self-similar profiles that may in turn be used to construct nonsimilar solutions like the ones that emerge from velocity profiles with finite boundary-layer thicknesses. Vaglio-Laurin et al.[(41)] have shown that it is possible to find an analytic solution for the shear and velocity distributions that Denison and Baum[(22)] obtained by a finite difference method. In addition they show how this analysis could be used for turbulent flow, provided that the eddy-diffusivity could be assumed a function of the streamwise coordinate alone.

To study the influence of the boundary layer in the region of the shear layer, Kubota and Dewey[(23)] assumed boundary-layer profiles of different shape with finite boundary-layer thickness at the corner, and found solutions using a momentum integral method. They divide the free shear layer into two parts, one above and one below the dividing streamline, and they use separate polynomial or exponential profiles to represent the velocity in each part. These profiles are matched at the dividing streamline, and closed form solutions of the momentum equation result. In particular, a quadratic and an exponential profile are investigated. The quadratic profile is practically the same in general behavior as the

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[†]Private communication by A. Goldburg. These are solutions within the Stokes-Oseen approximations.

Blasius profile, and the integral technique gives, asymptotically in the limit of the coordinate $\xi \to \infty$, the Chapman value for the dividing streamline velocity (0.587). However, for smaller values of ξ, the values of the velocities at the dividing streamline computed by Kubota and Dewey are appreciably higher than the numerical values obtained by Denison and Baum.[22] For a given value of the dividing streamline, the values of the coordinate ξ computed by Kubota and Dewey and Denison and Baum may differ by as much as a factor of 6. For this reason, and in view of the fact that the quadratic profile does not match the exact results of Denison and Baum for finite values of ξ, no conclusion can be drawn as to whether the higher dividing streamline velocities obtained for the same ξ, in the case of the exponential initial profile, are due to the profile itself or to inaccuracies caused by the integral method. The work is valuable, however, in showing that the integral methods, based at least on profiles with algebraic forms similar to the ones used by Kubota and Dewey, are unreliable in the case of the free shear layer problem.

In Refs. 42-44, Wan investigates the recirculating region for blunt and slender axisymmetric bodies through an integral method, in order to determine the location of the rear stagnation point and also to obtain initial conditions in the neighborhood of the neck for the further treatment of the diffusing core. He makes the assumption that the growth of the shear layer is linear in the direction of the mainstream. This is justified from shadowgraph pictures of wake flow in the base region, which seem to show a viscous core decreasing conically downstream from the base to the neck region. Both laminar and turbulent flows are considered. From his solutions, assuming a uniformly cool base, Wan finds that at the rear stagnation point one would expect to find a "cool core" surrounded by a shell of hot gas.† The theory suffers from the fact that experiments are used to make the equations soluble. One can argue that it is difficult to infer with certitude shear layer thicknesses that depend on the experimental device for their visualization. On the other hand, as shown by

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†This is a result of Wan's use of the Crocco integral, a relationship between temperature and velocity. This integral is an exact solution of the energy equation for Prandtl number one and zero pressure gradient.

the work of Denison and Baum and others reported above, the shear layer thickness can indeed be computed.

Lees and Reeves in Ref. 45 develop an integral method for laminar boundary-layer shock wave interaction; they show that in certain regions of the flow the results are quite dependent on the proper choice of the velocity profiles selected to represent the integral properties of the viscous flow. They find that polynomials are not successful, but wakelike solutions such as Stewartson's[46] reversed-flow profiles are. In Ref. 47 Reeves and Lees use the same method for the treatment of the laminar flow in the near wake of a blunt body. Their theory is capable of predicting the dependence of the base pressure and turning wake angles on the Reynolds number, along with the location of the rear stagnation point and the length of the recompression region around the neck. The theoretical trends are substantiated by available experiments behind cylinders.

The recompression region has also been studied by Webb et al.[48] using the momentum integral method developed in Ref. 45 by Lees and Reeves. The difference in their approach lies in the fact that they use polynomial profiles rather than Stewartson's similarity ones. Their results are in good agreement with the ones obtained by Reeves and Lees[47] who used Stewartson's profiles. Their findings indicate that in the recompression region where only one characteristic length determines the flow (the width of the wake), the solution is not very sensitive to profile shape near and beyond the rear stagnation point; the contrary is true for the flow very near the base of the body where two distinct characteristic lengths are present, namely, the thickness of the boundary layer shed from the body and the body base diameter.

From an experimental point of view the hypersonic wakeflow has been investigated only to a limited extent. Larson et al.[49] have probed the base flow of two-dimensional bodies at Mach number equal to 3. Also McCarthy,[50] McCarthy and Kubota,[51] and Dewey[52,53] have conducted experiments behind circular cylinders and wedges at Mach number equal to about 6.0, whereas Muntz and Tempel[54] report measurements at Mach numbers 13 and 18. Mentior should also be made of the very recent experiments of Todisco and Pallone,[55] undertaken behind a 10 deg half-angle wedge and a 10 deg half-angle cone at Mach number 16.

The investigation of wakes behind three-dimensional bodies is difficult because of the interference resulting from their support in wind tunnels. It has been conclusively shown by Dayman[56-58] that the support interference for spheres and cones flying at hypersonic speeds is considerable. For instance, during the free flight of a sphere at Mach number 3, the neck was formed 1.2 diameters behind the rear stagnation point; when the same sphere under the same flow conditions was supported with vertical wires 0.020 in. in diameter, the same distance was found to be 0.2 diameter shorter. For these reasons it is obvious that for wind tunnel work and detailed wake probing behind three-dimensional bodies, the model must be suspended without the presence of supports interfering with the flow. Magnetic suspension offers this possibility, and much is expected from such experiments now under progress at Princeton University. The suspension system is described by Zapata and Dukes in Ref. 59, and some preliminary results for sphere wakes at Mach numbers 16 and 18 in helium are reported by Vas et al. in Ref. 60.

Before closing this section, mention should be made of the interesting review by Nash[61] of research in two-dimensional base flow.

C. Wakelike Similarity Solutions

It was pointed out by Stewartson[46] that Falkner-Skan similarity solutions exhibit wakelike behavior if solved with boundary conditions that make the velocity at the wall finite and negative with zero slope, that is, with the boundary conditions $f(0) = f''(0) = 0$ and $f'(\infty) = 1$ and for a pressure gradient parameter β between 0 and $-\frac{1}{2}$. $f(\eta)$ is the familiar velocity function in terms of the similarity parameter η of boundary-layer theory.

In a series of more recent papers, Kennedy,[62] Stewartson,[63] and Kubota and Reeves[64] compute a number of cases and elaborate on the nature of these solutions. Kubota and Reeves deal with an axisymmetric case. All the papers contain the Chapman[21] constant pressure mixing solution as a particular case. The axisymmetric solutions yield a much larger reverse-flow velocity than do the two-dimensional solutions, presumably as a result of the axisymmetric geometry. A survey of the

literature on viscous free mixing with streamwise pressure gradients is given by Steiger and Bloom in Ref. 65, where a general class of linearized solutions is also obtained for two-dimensional compressible and axisymmetric incompressible viscous free mixing. In Ref. 66 a particular solution, which may be expressed in closed form, is discovered by Steiger among those formulated in Ref. 65.

As pointed out by Kennedy in Ref. 62, the pressure distribution in the case of the wake is much more complicated than the one normally used (a power law in the streamline direction), in order to comply with the existence of similarity solutions. Hence it is unlikely that a direct application of the wakelike results can be made. This is even more true for those flight regimes where the viscous-inviscid interactions are dominant and low Reynolds number effects cannot be neglected.

D. The Flow at the Neck and Farther Downstream

The geometric configuration of streamlines in the neighborhood of the rear stagnation point has been investigated independently by Cheng,[67] Kubota[68] and Vaglio-Laurin et al.[41] They all agree that the lines look like those shown in Fig. 1; this can be deduced from a Taylor series expansion of the Navier-Stokes equations for axisymmetric flow in the neighborhood of the rear stagnation point.

We now leave the recirculation region and examine the flow as it develops starting at the neck. It is obvious that the initial conditions at the neck are not known with any confidence since the free shear layer region still eludes analysis. One of the most important quantities needed is the centerline total enthalpy at the neck. The analyses reviewed by Baum et al. in Ref. 69, for example, give for a 10 deg cone at a free Mach number of 20 a ratio[†] h_D/h_e equal to 0.42 without expansion at the corner (that is, starting with a Blasius profile), whereas the same quantity with an initial isentropic expansion is equal to 0.?1. It is indeed difficult within the present state of the art for this region to defend with certitude any choice made for a "reasonable" initial velocity and corresponding enthalpy profile. In the analysis of Ref. 70 Pallone et al., recognizing

[†]Here D stands for dividing streamline and e for the inviscid external conditions.

this limitation, decide to use the Crocco integral in conjunction with an initial free shear layer velocity profile chosen from similarity solutions such as those of Cohen and Reshotko.[37] The Crocco integral, also used by Wan in Refs. 42 and 43 and by Webb and Hromas in Ref. 71, leads to a hot shell of gas with a cool core at the center, a familiar distribution in supersonic axisymmetric boundary layers. The authors themselves express serious doubts on the validity of the above procedure, but they attempt to justify it from the fact that the assumptions are verified a posteriori when the results are shown to be in good agreement with experimental values of wake thickness and its Reynolds number dependence. Their theory does not allow interaction of the core with the inviscid region; as one would expect, this leads to difficulties at high altitudes (low Reynolds numbers), where the viscous-inviscid interaction becomes more important. The study of Pallone et al. in Ref. 70 is made for wakes of pure air in thermodynamic equilibrium, or for completely frozen flow with a constant Prandtl number of 0.72. The wake is divided into an arbitrary number of strips in the streamwise direction, and the equations are integrated across the strips in the radial direction. The numerical solutions must start slightly away from the rear stagnation point since the equations are singular at that point. Nonsimilar velocity and temperature profiles are computed for a 10 deg semivertex angle cone and a cylinder. The size and flight conditions are so chosen as to match available experiments. Although they do agree with the experimental values[50] of the velocity decay behind a cylinder and wake thickness at the neck, the latter is not a strong test, since it is only a test of the conservation of the mass in bulk. It is furthermore shown in Ref. 70 that similarity solutions[72] of the wake are appropriate only asymptotically and are particularly inaccurate for the wake of a slender axisymmetric body that has a large velocity defect persisting far behind the body. In general, the velocity decays much faster in the nonsimilar region close to the neck than similarity theories predict; this is depicted in Fig. 9. Experiments to be described later show that for a certain range of Reynolds numbers the flow in the diffusing core from the neck can be laminar for considerable length. Hence the results of the above paper can be appropriately used in an attempt to correlate transition data for slender bodies, a subject to be discussed later.

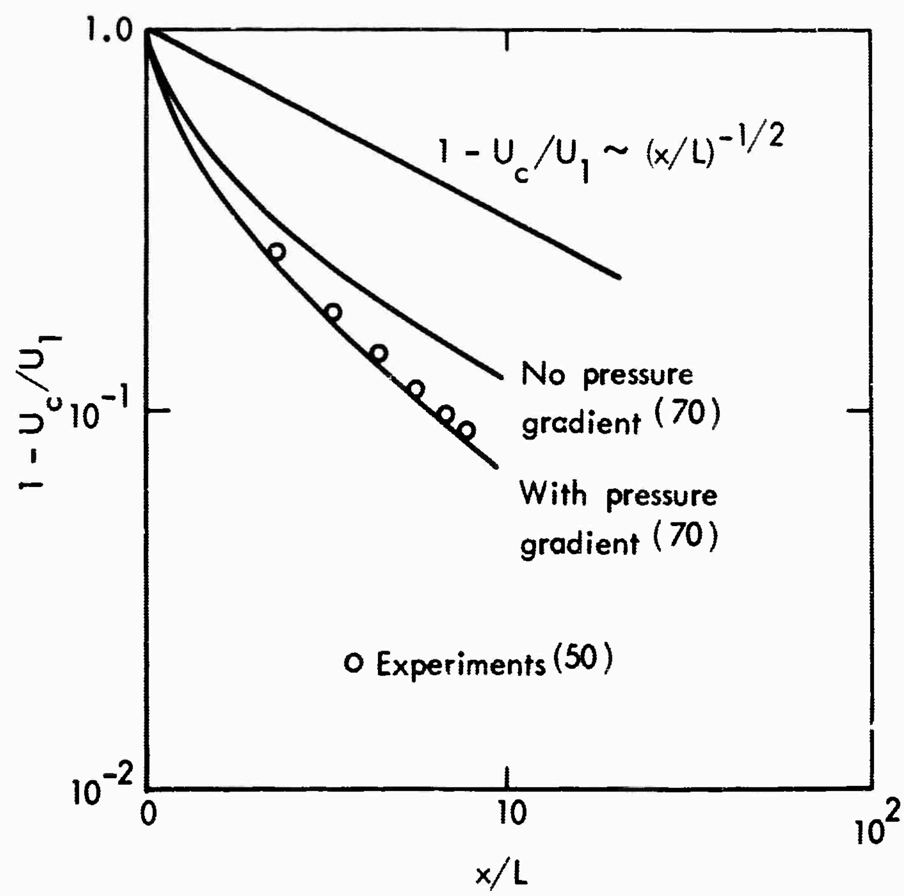

Fig. 9—Velocity decay along the axis in the laminar wake behind a cylinder: a comparison between experiment and theoretical calculations

In a further paper[73] Erdos and Pallone present numerical results obtained through their multistrip integral technique, which allows a quick determination of the laminar wake as it develops down the neck. This technique is particularly fit for the interpretation of experimental wake transition data. Here again no air chemistry is included in this work.

III. TRANSITION TO TURBULENCE

The first experimental study of transition to turbulence in the wake of a body moving with hypersonic speeds is perhaps due to Slattery and Clay.[74] This work has been followed by a large number of experimental studies, most of them using hypervelocity ranges[†] and a smaller number from the wind tunnel.[‡] The studies have been conducted with spheres, blunted cones, cones, cylinders, wedges, and other shapes. The measurements span a large range of free flight conditions in terms of Mach number, ambient density, and, to a smaller extent by necessity, characteristic lengths. In the free flight tests transition is identified with the aid of some kind of a photograph or film of the diffusing core; transition is identified as occurring at that distance behind the object in which some visual roughness is observed at the periphery or edges of the viscous core (the so-called intermittency region). This, of course, is not an objective criterion, but it is claimed that when a large number of persons look at these pictures they all agree very closely as to where turbulence starts. These measured transition distances to turbulence are normalized with respect to a characteristic length of the object (usually the base diameter), and the results are examined for their variation with Mach and Reynolds numbers. The difficulty in correlating the data appears to lie in the definition of these two numbers, a definition that should be guided by some theoretical understanding of the mechanism of transition. It is obvious that one should use the local laminar values in the region where turbulence occurs first, but this implies a theoretical and/or experimental knowledge that, as it has been mentioned previously, does not presently exist.

Webb et al.[91] give some order-of-magnitude theoretical considerations to the problem of transition based essentially on the early experimental results of Slattery and Clay.[75] They give the following description: A disturbance cannot reinforce itself at the wake axis if the difference of the velocity between the axis and the turbulent front

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[†]References 70, 75-88.

[‡]References 50, 51, 56-58, 89, 90.

is higher than the difference of the acoustic velocity at the same two points as pointed out by Lin.[92] Applying this condition at the neck, one can compute the above quantities in an approximate fashion, using the fact that $U_f/U_\infty \simeq 0.8$ for blunt bodies[†] and $U_f/U_\infty \simeq 0.99$ for sharp bodies. The local Mach number is found to be about 3 for blunt bodies, and about equal to the free stream value for very sharp bodies; hence transition for a blunt body will be closer to the neck than for a sharp body, primarily because of the well-known stabilizing effect of the higher local Mach numbers prevailing for sharp geometries. When transition occurs far downstream from the neck, it is recognized that a "wake Reynolds number," R_w, would be relevant to correlate the data. This Reynolds number is defined locally in terms of an average mass density, viscosity, the velocity deficiency between the front and the axis, and the width of the diffusing core. For an estimate of this Reynolds number, the Crocco integral is used along with Tollmien's asymptotic wake for the calculation of the velocity decay downstream and the growth of the core width. It is furthermore shown in Ref. 91 that $R_w \sim Re_\infty^{-\frac{1}{4}} p_\infty d$ for both blunt and sharp bodies, indicating that since the free stream Reynolds number dependence is weak, stability depends essentially on the product $p_\infty d$ alone. This conclusion has been also reached experimentally through the results of Slattery and Clay,[75] Demetriades and Gold,[89] and Hidalgo et al.[81] In terms of these data alone it was also found that for blunt bodies the Reynolds number based on the transition distance and conditions prevailing at the turbulent front has the value of 5.6×10^4, whereas for sharp bodies the value was higher by a factor of 4. As pointed out by Lees[93] and Zeiberg,[94] the value of 5.6×10^4 is consistent with the low Mach number data for transition in separated shear layers (see papers by Chapman et al.[27] and Larson[95]) and the incompressible flow data for wakes of flat plates (see the work of Sato and Kuriki[96]).

Gold[97] made a theoretical study of the dynamical (inviscid) stability of axisymmetric laminar wakes, essentially extending the work

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[†]See Eq. (8) and in particular Eq. (10) with the discussion that follows.

of Batchelor and Gill.[98] His numerical calculations are based on the assumption of Gaussian† far-wake velocity and static-temperature profiles. He shows that two-dimensional compressible wakes are generally dynamically unstable, whereas an axisymmetric disturbance mode‡ ($n = 0$) exists under special conditions, but the mode corresponding to $n = 1$ is always unstable. For the latter it is also found that the maximum amplification rate increases with increasing wake core temperature until a critical temperature is reached above which the maximum amplification rate starts to decrease. Furthermore, as the wake core temperature increases, the critical Mach number increases and the range of Mach numbers above which subsonic disturbances can exist also increases. Lees[93] presents the above results together with some estimates of the minimum critical Reynolds number below which wake turbulence cannot maintain itself against the action of viscous dissipation. These estimates are based on the crude assumption that as the ambient density decreases, the downstream movement of the transition point will stop where the effective turbulent diffusivity becomes smaller than the appropriate laminar diffusivity. A tentative picture of transition in the wake of hypersonic speeds is offered by Lees in Ref. 93 as follows:

> Below a certain minimum critical number the wake is laminar. Above this limit, transition first appears at about 40-50 diameters†† behind the body if the body is blunt, and then moves forward as the ambient pressure increases, maintaining a constant value of Reynolds number based on the local distance x and conditions prevailing at the turbulent front of 5.6×10^4, independent of body diameters. When transition reaches the neck, it gets stuck there until the Reynolds number based on the outer inviscid conditions exceeds the Reynolds number computed in the free shear layer; then it jumps to the

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†For the incompressible subsonic case with the same assumption, see Ref. 99.

‡In this notation the disturbance amplitude is proportional to exp (in ϕ).

††Lees' statement should be compared with the data of Clay et al.,[100] who indicate the possibility of transition well above this limit with transition observed at more than one thousand diameters for Mach numbers of the order of 17; however, before accepting these numbers, their statistical significance should be scrutinized. In fact, some recent experiments in a ballistic range by Wilson[101] for spherical models at Mach numbers in the neighborhood of 18 contradict the data of Clay et al.

free shear layer and eventually appears in the boundary layer on the body. For a sharp-nosed slender body, transition first appears in the wake at lower ambient pressure and somewhat farther back in terms of body diameter than for a blunt body, especially at lower hypersonic speeds. Again transition moves rapidly forward as the ambient pressure increases but this forward motion slows down when the location of transition approaches the neck.

In a qualitative way the above picture of transition to turbulence in the diffusing core of a hypersonic wake seems to be substantiated by experiments, but it is obvious that exact numbers that correlate transition lengths with some characteristic Reynolds and Mach numbers have yet to be settled, for the reasons enumerated earlier. A collection of observations related to transition to turbulence for bodies of different shapes is shown in Fig. 10, reproduced from Ref. 102.

Pallone et al.[83] have used their laminar wake results (discussed in Sec. II.B) to evaluate the local properties in the cross section where transition occurs. They show that a group of experimental data correlates in a Reynolds-versus-Mach number diagram when the physical properties entering these parameters are evaluated at the edge of the core, and when the characteristic length and velocity are taken as the width of the core and the velocity difference between the axis and the edge of the core.

An attempt to correlate existing experimental data[†] in a semiempirical way is due to Zeiberg.[102] The properties in the inviscid downstream wake region are estimated by assuming the flow history to consist of an appropriate "body shock" and a subsequent isentropic expansion to a free stream pressure. For a sphere the forward shock is considered to be normal, whereas for the various cone and wedge angles an appropriate conical or oblique shock is used. Perfect gas relations are used. The bluntness or body shape is taken into account by the ratio of the free

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[†]Levensteins[82] and Wen[103] are also among those who have concerned themselves recently with the same problem. One should also note the work of Birkhoff et al.,[80] a most interesting presentation of photographic data of wakes behind bodies with and without angles of attack. They made the interesting observation that there are cases in which the flow shows a turbulent core for some distance, which is followed by a laminar region, and then after a further distance becomes turbulent again.

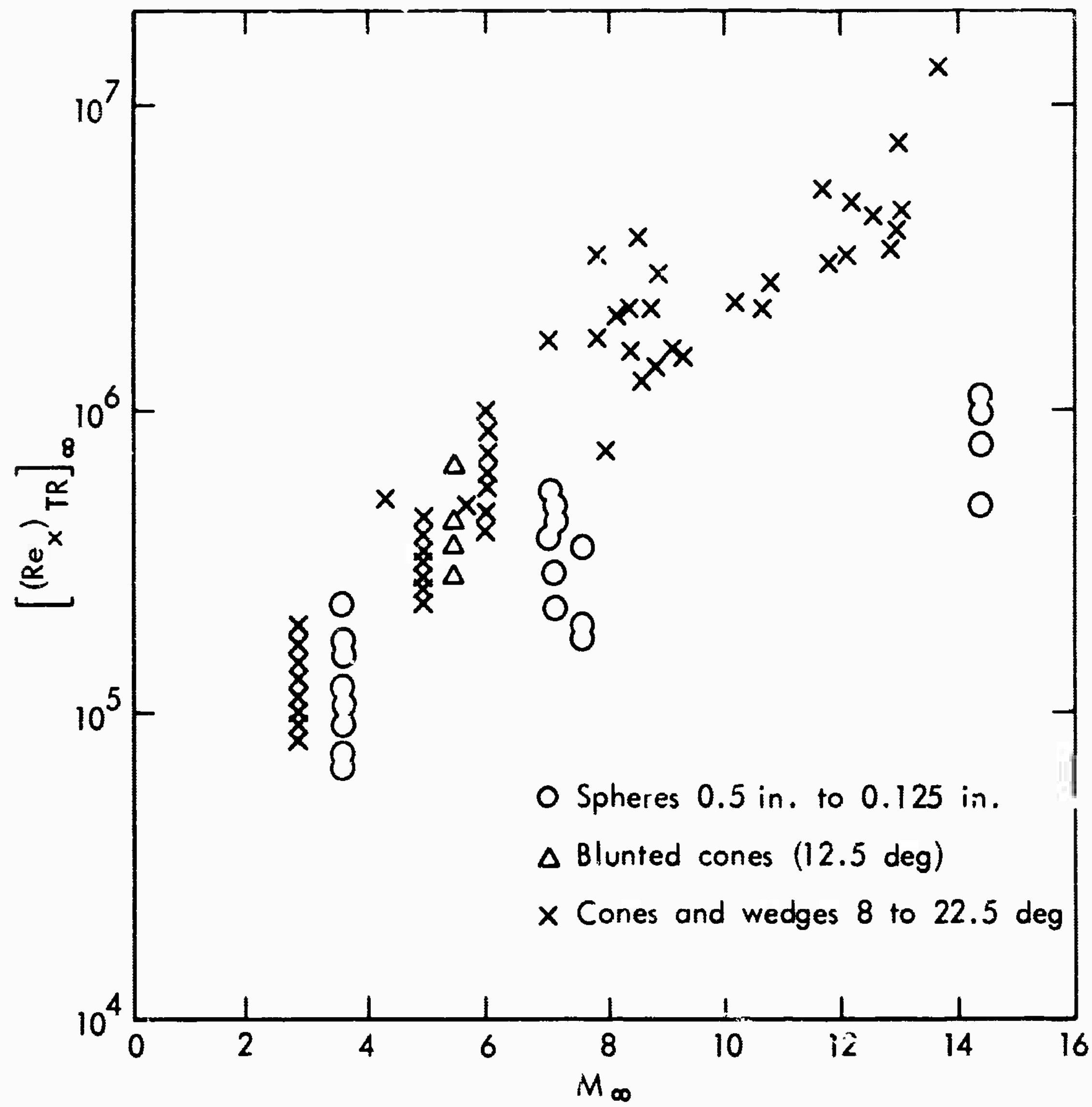

Fig. 10—Experimental data for wake transition[102]

stream Mach number to the local inviscid Mach number.[†] The correlation has the form of the product of the square of this Mach number ratio times the free stream Reynolds number based on the transition distance. When all the data are plotted (for cones, spheres, wedges, blunted cones, and cylinders), in terms of the above parameters as a coordinate versus the free stream Mach number (as shown in Fig. 11), these data seem to collapse into one group.[‡] The table incorporated in Fig. 11 is taken from Ref. 104 and gives the range and nature of the experimental data plotted in Fig. 11. It should be recognized that although theoretical concepts were considered, this correlation is empirical. In the same paper Zeiberg also made an attempt to find a correlation for the so-called sticking distance,[††] the point where for slender bodies transition "gets stuck" at some distance behind the body;[(93)] however, Zeiberg found that an empirical correlation was not warranted for the data and ranges available.

Zeiberg's "unified" correlation of transition is based on a Reynolds number using the transition length and not the thickness of the core as the characteristic length; also it depends on the velocity and not on the local velocity deficiency, contrary to what we expect based on our understanding of transition to turbulence.[‡‡] As pointed out by Erdos and Gold,[(104)] Zeiberg's correlation is not capable of showing the effect of body size on transition, contrary, for instance, to the findings of Pallone et al.,[(83)] who show this effect in their correlations based on theoretical computations of the laminarly diffusing core. This is shown in Fig. 12,

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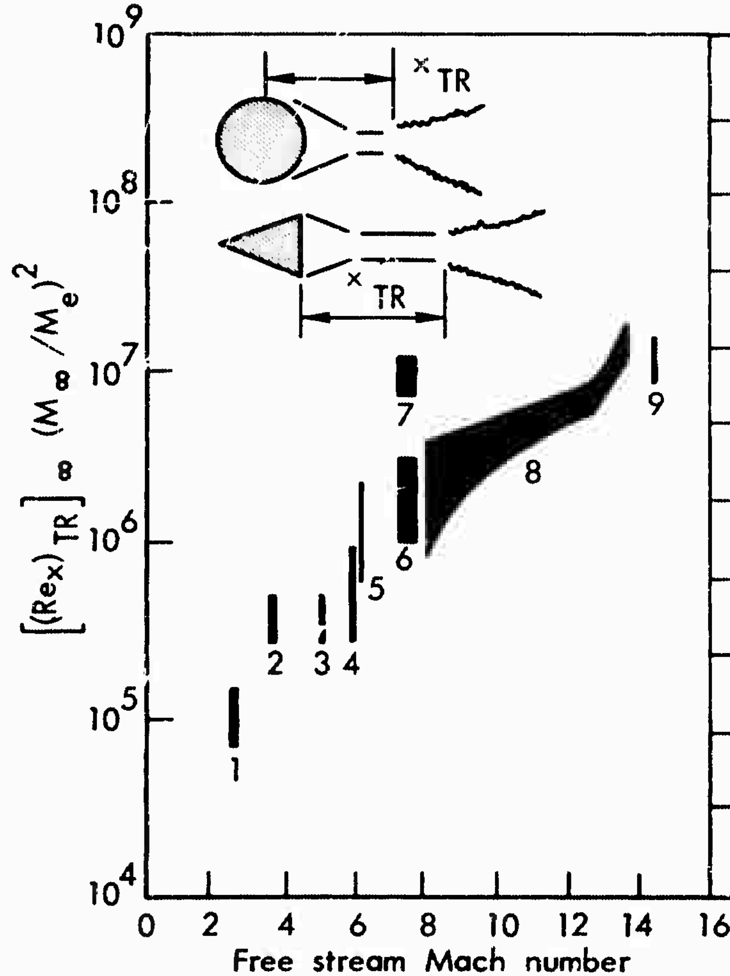
[†] As pointed out in the discussion of Eq. (10), nonequilibrium effects strongly affect this number; hence perfect gas relations can lead to considerable underestimates.

[‡] The local Mach number M_e used in this correlation is given versus the free stream Mach number showing the relative independence of M_e versus the free stream Mach number for spheres and cylinders on one end, and a straight line for the case of slender cones on the other.

[††] This term has been introduced by Demetriades[(105)] and arose from the fact that for a slender body the local Mach number is high at the neck and thus transition is inhibited, whereas at some distance downstream, where lower Mach numbers prevail, transition is possible. See also the stability arguments related to this phenomenon by Kronauer.[(106)]

[‡‡] Such as presented, for instance, by Webb et al.,[(91)] Pallone et al.,[(83)] and Gold.[(97)]



Label	Reference	Geometry	$(Re_d) \times 10^{-5}$	$(x/d)_{TR}$
1	115	Cone	0.026 - 0.23	3 - 30
2	114	Sphere	0.10 - 0.80	2 - 25
3	115	Cone	0.026 - 0.89	5 - 42
4	211	Wedges	0.091 - 0.72	6 - 74
5	28	Cylinder	0.10 - 0.45	4 - 56
6	114	Sphere	0.29 - 3.0	2 - 25
7	114	Sphere	0.21 - 0.45	54 - 100
8	105	Cone	0.85 - 8.5	4 - 22
9	114	Sphere	0.43 - 1.7	5 - 35

The cones are of a semivertex angle, 10 - 15 deg
The wedges vary between 2.5 and 22.5 deg

Fig. 11—Zeiberg's correlation(102)

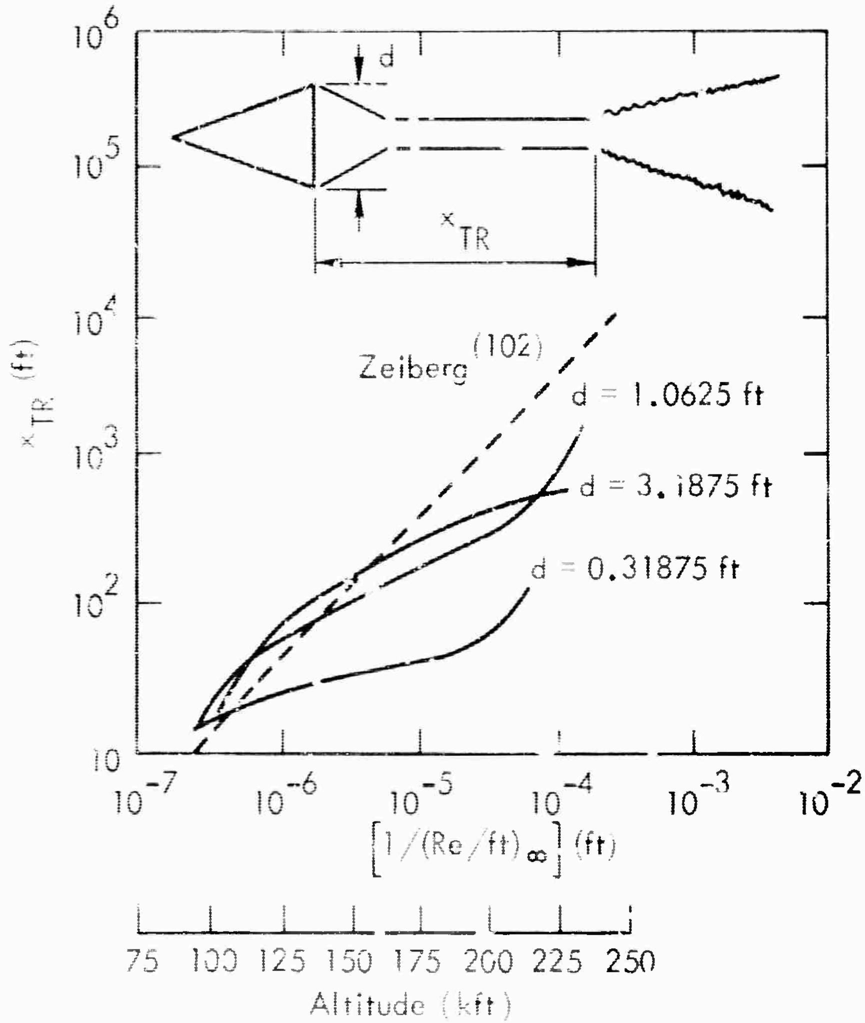


Fig. 12—Comparison of Zeiberg's correlation[102] and the theoretical results of Pallone et al.[70]

where the transition distance is plotted versus the inverse of the free stream Reynolds number for a 12 deg cone flying with a speed of 22,000 fps, and for different cone sizes. For this reason Zeiberg's correlation should be taken as an interpolation formula valid within the range of experimental data he used; these are limited, as pointed out earlier, by small body sizes. It should be recognized, however, that the correlation still has the advantage, as long as it is valid, to offer a quick computation of nondimensional numbers based essentially on values taken at the free stream, and from a graph to determine the transition distance, if it exists.

IV. TURBULENCE

From a theoretical point of view, very little seems to be known about shear turbulence. Whatever is known with confidence is the result of detailed experimental work. In the case of the turbulently diffusing core formed behind an object moving at hypersonic speeds, detailed experimental information is not as yet available, and therefore we must rely on extrapolations from what is known for subsonic incompressible wakes. The first systematic attempt to do so is due to Lees and Hromas.[10] Their theory was compared with the experimental data of Slattery and Clay,[74] which exhibit the growth of the turbulent core behind spheres at hypersonic velocities. Since the work of Lees and Hromas, data relating the growth of the turbulent front with axial distance have come from many investigators[107-113] for different geometries and flow conditions.† A great variety of computations have been performed in the turbulently diffusing core, which include the effects of chemistry for configurations ranging from cones to spheres with different degrees of bluntness.‡ A number of experiments have also been performed measuring the axial electron density behind hypersonic wakes.[111,132-135] Attempts have been made to correlate the theoretical results with experimental work yielding the electron decay in the turbulent core[93,126,136] and also the velocity decay downstream.†† Experimental[100,105] and theoretical work[137-140] has also been undertaken for the study of the relation between electron and mass density fluctuations, a knowledge very much needed for the computation of the interaction of an electromagnetic beam with the ionized wake. So that we can review the work mentioned above in an orderly fashion, we shall deal with it in separate sections: (A) Blunt Bodies, (B) Slender Bodies, (C) Chemistry, and (D) Fluctuations.

A. Blunt Bodies

Turbulence is always associated with regions in which high vorticity

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†References 75, 76, 79, 85, 87, 107-113.

‡References 71, 93, 114-131.

††References 78, 81, 83, 108.

is present. The free shear layer formed after separation occurs is, of course, such a region. We shall assume that in the diffusing core near the neck turbulence is present, which will engulf the fluid mass that has gone past the blunt body into the inviscid part of the wake. During the first few diameters downstream of the neck, the "engulfing" mechanism will be dictated solely by expansion, since just behind the trailing shock the temperature and pressure are very high. As the pressure drops to nearly ambient, turbulent diffusion will become predominant--swallowing up more and more momentum and finally spreading to a point where the whole momentum deficiency is governed by turbulence.

A calculation based on the above model requires the following a priori knowledge: (1) the position and thickness (in terms of mass, momentum, and energy) of the neck; (2) pressure profiles downstream of the neck; and (3) a model for the estimation of the rate of energy diffusion due to turbulence. The first two items presume a solution of the near-wake problem with a result describing the interaction of the trailing shock with the free shear layer, a rather difficult and largely still unsolved problem. As a rough approximation one may obtain some information for the first two items by solving the inviscid flow problem over a sphere followed by a backward facing cone, which takes the place of the shear layer, and then a cylinder fitted at the neck. Shadowgraph pictures can be used for guidance in prescribing the above geometry. Such calculations are reported by Lees and Hromas in Ref. 10. For item 3 the only choice offered is an extension of the subsonic eddy-diffusivity theories to the compressible case. Such an extension is provided by Ting and Libby[141] and Lees and Hromas.[10] Both references present an equivalent eddy-diffusivity $\mathcal{E}$, which at the centerline is given in terms of the incompressible eddy-diffusivity ϵ as

$$\mathcal{E} = \left(\frac{\rho(0)}{\rho_f}\right)^{2/(1+j)} \epsilon . \tag{13}$$

In the above, $\rho(0)$ indicates the mass density at the axis, and ρ_f the density at the "front" of the turbulent core; j assumes the value zero for the two-dimensional case and one for the axisymmetric case. Now ϵ is proportional to the velocity difference between the turbulent front[142]

and the axis, and hence it is also proportional to the drag coefficient corresponding to the momentum deficiency in the turbulent core, which is variable in the stream direction.† Lees and Hromas[10] therefore suggest that experimental evidence should also be used in deciding between the two extreme cases--a diffusion coefficient that is proportional to a constant frozen drag (say, the one at the neck), or one that is proportional to the local drag.

In the same paper Lees and Hromas attack the problem by assuming that the inviscid-enthalpy profile prevailing in the laminar region is known; inside the turbulent core a parabolic temperature profile is assumed that extends up to a distance where the turbulent front appears, which is one of the unknowns of the problem. In this analysis the momentum equation is not satisfied, but in its place the assumption is made that the velocity deficiency is known. In this respect it can be assumed that the ratio of the velocity at the center divided by the velocity prevailing at the turbulent front is equal to 0.8, as suggested by Eq. (8), and that it remains constant downstream. On the other hand, the energy equation is satisfied only along the axis, whereas in the radial direction energy is conserved in bulk between the centerline and the turbulent front, after the fashion of integral methods. The eddy-diffusivity of Eq. (13) is used where ϵ is defined by its incompressible definition;[142] it is kept in mind, however, that this definition is strictly valid only in the neighborhood of the axis of symmetry. The calculations start with an assumed value of both the enthalpy at the center of the neck and the drag coefficient corresponding to the neck thickness. Thermodynamic equilibrium is also assumed along with ideal gas relations. The resulting equations, written in the Howarth compressible plane, are integrated downstream numerically, where the "laminar" conditions prevailing at the turbulent front are kept unaltered, since an order-of-magnitude argument shows that the laminar decay is by far slower than the fast engulfing turbulent mechanism. Two such calculations are performed in Ref. 10: one in which the value of ϵ is supposed to assume instantaneously the local value, and the other in which ϵ is essentially frozen at the initial

†In contrast to the subsonic case, where the turbulent core includes all the drag.

conditions. The experimentally determined growth of the core favors the former point of view, which is the case of the self-preserving wake. This assumption is also equivalent to stating that the local intensity of turbulence is proportional to the corresponding value of the velocity deficiency and that the scale of turbulence is proportional to the local width of the core. The value of the velocity deficiency times the width divided by the eddy-diffusivity is then a universally constant Reynolds number (R_T). It turns out that if the number's value is chosen to be the same as the one suggested by the work on subsonic wakes ($R_T \simeq 14$),[143] the theoretical growth predictions are found in good agreement with the experiments.†

In Ref. 9 Lykoudis used the same physical model as the one adopted by Lees and Hromas;[10] however, his mathematical approximations differ, and yield closed form solutions rather than numerical ones.

The main results of the above analyses for the case of spheres are the following: The rates of growth for the turbulent core are higher for higher Mach numbers and higher values of the enthalpy at the neck.‡ The effects of both Mach number and neck enthalpies disappear at about one thousand diameters downstream when the core changes its growth rate to vary as the one-third power of distance multiplied by the drag coefficient and the body cross section.†† For the first few diameters downstream of the neck, the growth is dictated by pure expansion; see Eq. (2). Later it is obvious that an inflection point occurs in the curve of growth versus distance. This can be identified as the point where the swallowing process in the wake has reached the change of slope in

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†Kronauer in Ref. 106 reexamines the value $R_T \simeq 14$, which he accepts as a lower limit. He suggests that for the experimental data reported by Birkhoff and Eckerman[144] and Slattery and Clay[78] the value of R_T is within the limits $24 < R_T < 36$.

‡In Ref. 10 Lees and Hromas choose the value of the fraction of the stagnation enthalpy to which this enthalpy is equal to be about 0.5, whereas Lykoudis in Ref. 9 makes a study using this fraction as a free parameter. The value of 0.5 seems to be correlating the growth data rather well. In Ref. 83 Pallone et al. choose a value equal to 0.3 for best fit of theory with electron density measurements for a cone geometry.

††This, of course, is the incompressible behavior of the wake that one would expect as an asymptotic value.

the radial laminar enthalpy profile, which presumably describes the region outside the turbulent core. This behavior is exhibited in Fig. 13, where the theory is compared with some experimental data.

The scatter of the data on wake growth is unfortunately so large, especially in the critical region close to the neck, where the compressibility effects are expected to be the highest, that it is difficult to accept as proof of the success of the theory the fact that the theoretical curves seem to pass through the points. A more important confirmation of the premises on which this theory rests would be a comparison with more sensitive quantities, such as temperature decay and electron concentration. Unfortunately, temperature data are not available, whereas the more or less abundant experimental data giving the electron decay are dominated so much by chemistry that the correctness of the eddy-diffusivity model cannot be assessed.

In a recent paper Zeiberg and Bleich[136] examine the same problem with a finite difference method for the turbulent core and one-dimensional inviscid streamline equations for the outer inviscid wake. In this analysis they include nonequilibrium air chemistry. As a model for eddy-diffusivity, they adopt the one proposed by Ting and Libby.[141] Zeiberg and Bleich assume that there is no interaction between the core and the inviscid region, and that the pressure distribution in the turbulent region is given by the first-order blast-wave analysis. Their calculations start with a given initial viscous and inviscid cross section; unfortunately, the details of the initial profiles are not spelled out in the report. The inviscid conditions prevailing in the outer flow are approached at infinity, as far as concerns the turbulent boundary-layer solution of the inner flow. The "edge" of the turbulent core is determined, as is appropriate in boundary-layer-type solutions, by stating that it is reached at the point where the velocity has approached a prescribed percentage of the inviscid velocity. Their results show the same qualitative behavior of growth as does the integral method presented in this section, but in general the wake seems to be somewhat thicker than experimental data suggest[79,109,145] for M = 5.05 - 8.5 and one atmosphere pressure. The one-third power growth law, however, is reached shortly after one thousand diameters, which agrees with the integral methods. The decay of the enthalpy along the axis is not given

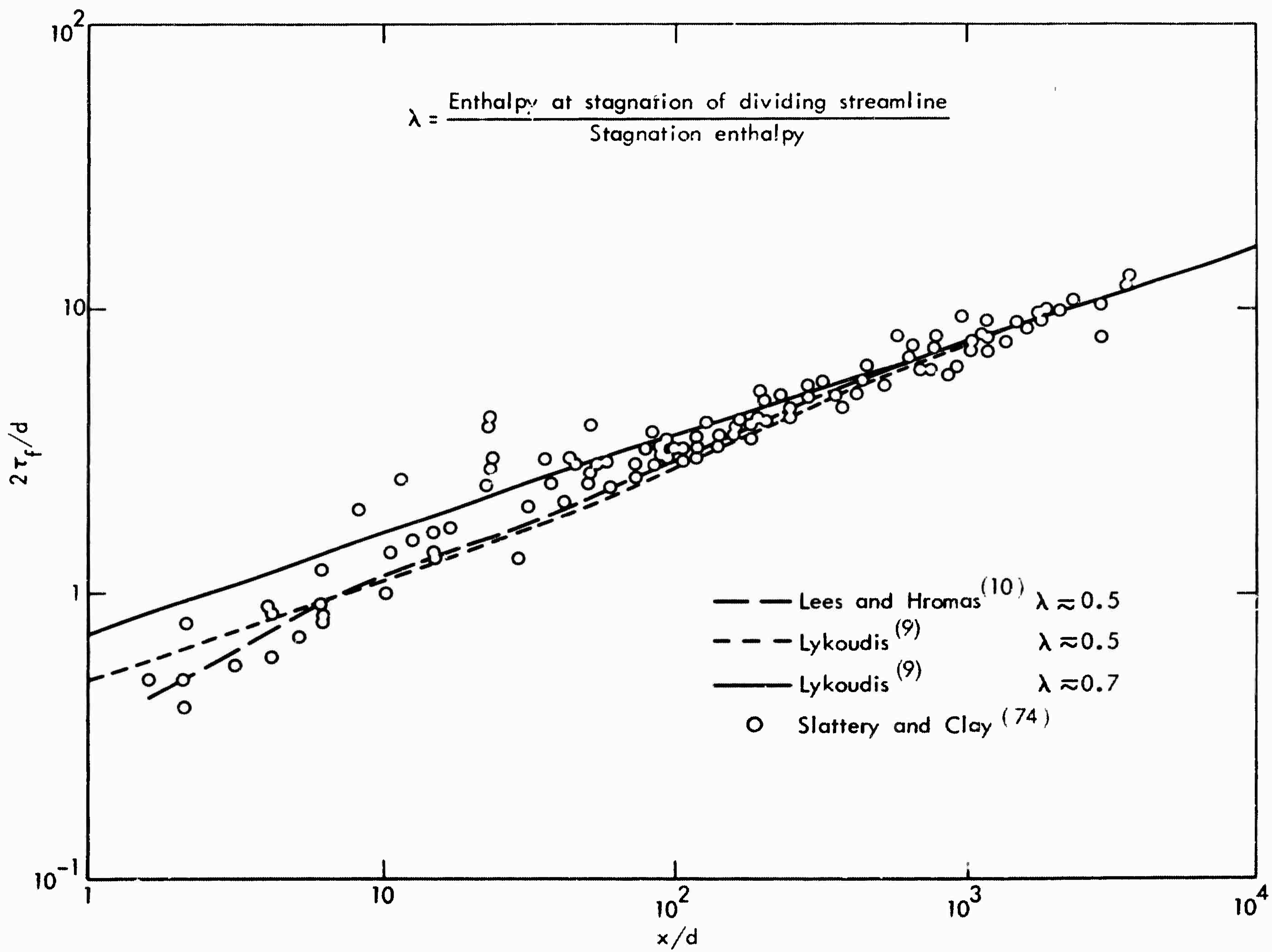

Fig. 13—Comparison of ballistic range experiments and theoretical predictions for the growth of the turbulently diffusing core behind a sphere moving at Mach = 8.5

in Ref. 136, and so no comparison can be made between the results of the integral method and the numerical one for this parameter. However, an electron density comparison is made with experimental results, and this point will be discussed in Section C.

B. Slender Bodies

Slender and blunted slender bodies have been studied extensively both from the theoretical and experimental point of view. The theoretical methods are not different in principle from those presented in conjunction with blunt spherical bodies. For instance, Hromas and Lees[146] extend their computation of Ref. 10 to the case of pure cones and blunted cones with different drag coefficients under the assumption that the flow is in thermodynamic equilibrium. Lykoudis[9] also provides a similar extension of his sphere computations. Both papers use an integral technique with a parabolic profile for the core enthalpy, but there is a large collection of papers in which the problem is handled with more complicated integral approximations and numerical techniques and/or different assumptions for the eddy-diffusivity.[†]

The basic difference between the case of the turbulent wake behind a blunt body, such as analyzed in the previous section, and that behind a slender one is that in the first geometry high temperatures are developed at the forward stagnation point, whereas in the second high temperatures are developed in the boundary layer, where the flow can only come to rest through the action of the viscous forces that act parallel to the wall. For blunt bodies, it was found that most of the momentum deficiency was located in the inviscid flow, because of the action of the strong shock wave and a high characteristic enthalpy prevailing at the cross section where the pressure becomes ambient; as mentioned previously, this enthalpy is equal to about one-third the stagnation enthalpy. The distribution of the drag at the neck was such that the total drag coefficient was much higher than the drag coefficient associated with the neck. Consider now the extreme case of a thin cone. Even at high Mach numbers the oblique shock wave formed at the tip barely decelerates the flow, and hence the inviscid

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[†]References 7, 41, 114, 115, 122-125, 127-130, 147, 148.

enthalpies are very low compared with the enthalpies prevailing inside the boundary layer or at the neck. In this case one expects to find a thicker neck. The development of the turbulently diffusing core will then depend to a very large extent on the relative magnitudes of the inviscid and viscous drag. The turbulent calculations can start at the cross section where turbulence is first observed if the conditions prevailing there are known from laminar calculations.

On the basis of the results of Hromas and Lees[146] and Lykoudis,[9] the following conclusions can be reached: The level of the enthalpy at the neck is of the same order for both blunt and slender bodies,[†] since at the neck there is a tendency for the enthalpy to be recovered at the point where the dividing streamline stagnates. However, boundary-layer effects will be more prominent for slender bodies. In the case of cones or blunt bodies, the smaller the amount of total drag, the higher the decay of the enthalpy. Furthermore, for the case of a blunt body with high drag, cooling is possible through the mechanism of expansion, a mechanism practically absent in the case of a slender cone. Expansion cooling for the flow around blunt bodies competes with the reduced drag mechanism for the slender bodies, but the end result is that the enthalpy decays faster for slender than for blunt bodies.

For the case of a blunted cone, overexpansions and recompressions are to be expected; the initial enthalpy profile will be similar to a Gaussian distribution but with more inflection points, and as a result the growth of the turbulent core will be "wigglier" than the growth of a pure cone. At the same time, even though both a cone and a blunted cone might have the same overall drag coefficient, the blunted cone will have a thicker core initially, although after a sufficient distance both growth curves will eventually collapse to the same one-third incompressible law. These effects are shown in Fig. 14, taken from Ref. 146.

A more refined technique for investigation of the turbulent core has been proposed by Zeiberg and Bleich.[129] With the use of a large computing program they have been able to find finite difference solutions

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[†]From the best estimates available today, the ratio of the neck enthalpies for a blunt and slender geometry is equal to 0.5/0.3, as discussed before.

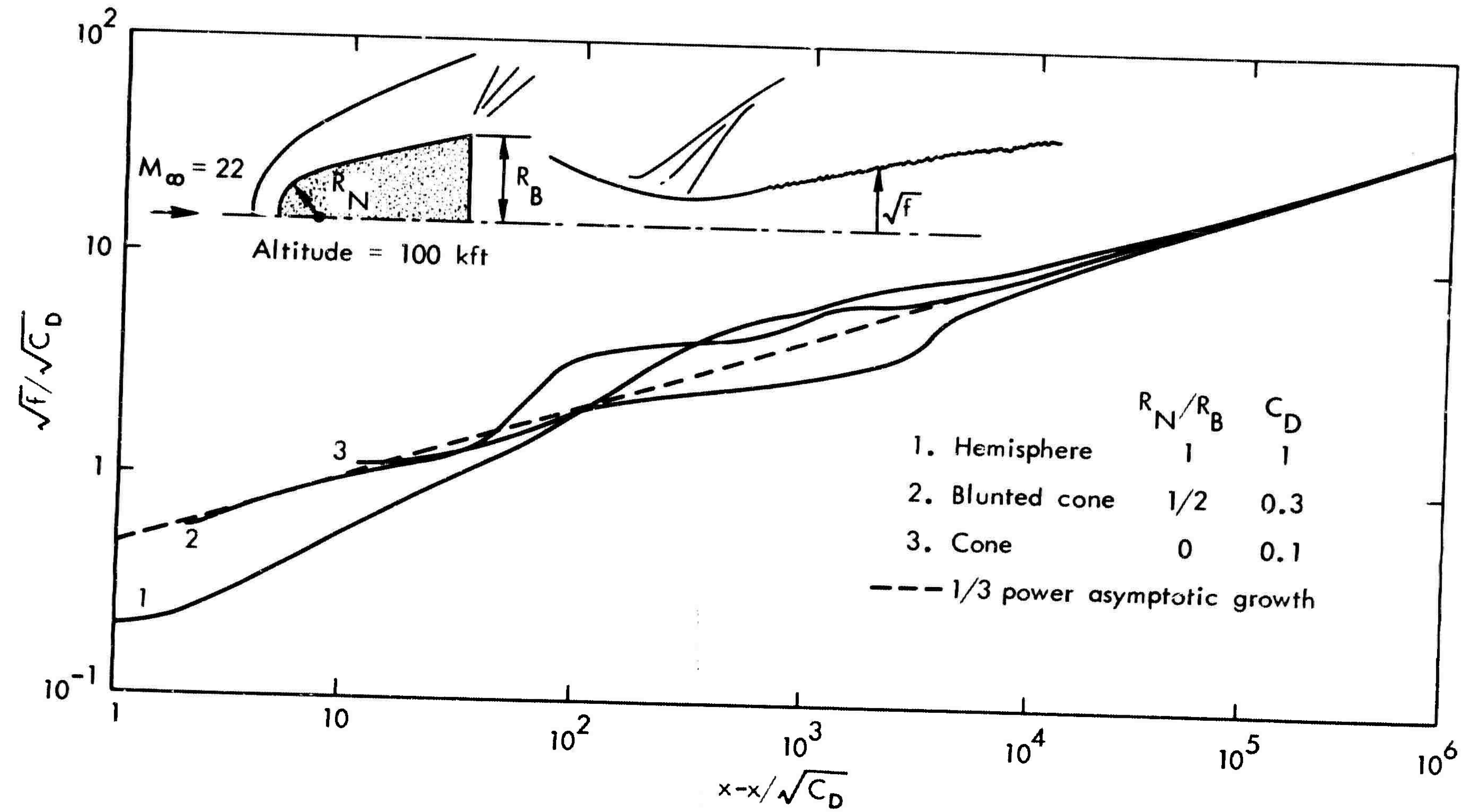

Fig. 14—Theoretical studies of the effect of the bluntness on the growth of the turbulent wake

satisfying all the pertinent conservation equations everywhere in the flow field. The obvious limitation of such a method lies in the fact that only one numerical answer is obtained for each specific geometry and flow condition; the advantage is that it offers the opportunity to compare the answers with more general results obtained by uncertain integral methods and hence check their validity. Such a check was first undertaken for the laminar case† between the rough integral solution of Bloom and Steiger,(148) essentially a one-dimensional treatment, and the finite difference method, when the same initial profiles are used. It is noted that although the integral method correctly predicts the quantitative behavior of the flow variables, the qualitative differences can be substantial. These differences are greatest for the axial temperature profile and are more prominent at the higher altitudes. (The comparison was made at two altitudes, 150 kft and 250 kft.) The effects of Lewis and Prandtl numbers when they are different from unity was also studied and found to be prominent. In the turbulent case the finite difference study is undertaken for four different models of eddy-diffusivity. The first model, used by Bloom and Steiger,(148) differs essentially from the one used by Lees and Hromas(10) and Lykoudis(9) in that the mass density is evaluated at the center rather than at the edge of the core. The second model, due to Ferri et al.,(150) combines the mass density with the velocity in a product, so that instead of making the eddy-diffusivity proportional to the mass and the velocity deficiency separately, it makes it proportional to the difference of ρu evaluated at the edge and at the center. The third model is the one of Ting and Libby;(141) the fourth is the same as the third, except that it is applied everywhere in the core with the value it has at the center, precisely as was done in Refs. 9 and 10. The finite difference calculations were made for a 10 deg half-angle cone at a velocity of 23,000 fps. Two altitudes were considered, one at 100 kft (where the flow is in thermodynamic equilibrium) and one at 120 kft. The initial enthalpy at the axis was assumed to be one-third of the free stream stagnation enthalpy. The main

†Extensive calculations for this case are reported in Ref. 149 by Zeiberg and Bleich.

result of these calculations is that the third and fourth models give practically the same results, whereas the first and second models give results one order of magnitude smaller and larger for the axial temperature and velocity distribution, respectively. In Ref. 151 Wen studied independently the problem of different definitions for eddy-diffusivities and found the same qualitative results as Zeiberg and Bleich[(129)] by making use of an integral technique.

C. Chemistry

There seems to be general agreement among the different workers as to the rate constants for chemical processes involving air at high temperatures. These data refer to time-independent conditions. No conclusive study is available to indicate the deviations from these rate constants due to turbulent temperature fluctuations.[†] The common sources of references are Nawrocki,[(153)] Lin and Teare,[(154)] Bortner,[(155)] Bortner and Golden,[(156)] Chanin et al.,[(157,158)] and Teare and Dreiss.[(159)] Comparisons among the different results are difficult even with the same chemistry, because the initial conditions are not the same. In the only case known, however, in which four laboratories worked independently with the same initial conditions, they all agreed as to the general features of the turbulent core when chemistry is taken into consideration.[‡]

The influence of chemistry in hypersonic wakes has been considered in a number of early reports by Lin,[(5)] Lin and Teare,[(160)] Goulard and Goulard,[(6)] Bloom and Steiger,[(114, 115, 148††)] and others. More recent contributions in this area are the papers by Li,[(123, 124)] Zeiberg,[(131)] Lien,[(120)] Lien et al.,[(121)] Webb and Hromas,[(71)] and Lin and Hayes.[(140)] Lees[(93)] has also made an order-of-magnitude analysis of the effects of chemistry on the flow properties and in particular the decay of electron

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[†]Some preliminary work relevant to this problem is reported by Eschenroeder in Ref. 152.

[‡]The four laboratories were the General Electric Company, Space Technology Laboratories, Avco Corporation (Research and Advanced Development Division), and General Applied Science Laboratories. The project was suggested by the Institute of Defense Analysis, and the results were reported by Menkes at the April 1964 AMRAC meeting.

[††]Ref. 148 is the first full-scale analysis of the nonequilibrium chemical aspects in hypersonic wakes.

density due to the mechanism of oxygen attachment. Webb and Hromas[126] have performed similar computations by using an integral technique, which, although not as accurate as some of the more refined integral methods or purely numerical ones, has the advantage in some cases of yielding direct analytic solutions and therefore provides a window from which the essence of the complicated chemistry effects become more apparent.

In the two extreme cases of a sphere and a pure cone, the chemical kinetics of the sphere are by far the more complicated because of the large temperatures behind the bow shock wave and the resulting fast expansion. Several workers have established that for a blunt body, 1 ft in diameter, the inviscid flow is in thermodynamic equilibrium below the altitude of 100 kft, whereas at an altitude above 150 kft the assumption of "frozen flow" is justified.[161-163]† For reentry velocities of less than 25,000 fps, one also finds that the momentum history of the flow is affected negligibly by the chemistry. This holds both in the regions around the body and in the wake behind. In this way the chemical aspects of the diffusing wake core can be examined by assuming that the width of the core is the one derived assuming chemical equilibrium.

In the case of a blunt body, a stream-tube method is commonly used for the study of the inviscid chemistry starting with the forward shock; a pressure distribution resulting from a thermodynamic equilibrium calculation is assumed. As indicated earlier in this memorandum, for the evaluation of the pressure distribution in the free-mixing layer and the trailing shock region, a cone and a cylindrical stem are assumed to exist (with angles and thicknesses suggested by, say, schlieren photographs) so that the pressure is obtained everywhere in the flow field. Such calculations can be found, for example, in Ref. 160 by Lin and Teare, Ref. 148 by Bloom and Steiger, Ref. 122 by Lenard et al., and Ref. 71 by Webb and Hromas. When the electron density is very high and the mass density rather low, the mechanism most effective for the disappearance of electrons is the NO dissociative recombination: $NO^+ + e^- \rightarrow N + O$;

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†See, as an example, Lin and Hayes in Ref. 140 for a graph of the approximate boundaries for scaling of chemically reacting hypersonic flow behind spheres at 22,000 fps velocity in air.

this reaction is important near the body where the above conditions prevail. The electron diffusion equation for this reaction states that the rate of disappearance of the electrons is proportional to the square of their concentration and, assuming constant velocity and an average temperature, we find upon integration that away from the body the electron concentration decays with the inverse power of the distance along the axis.[160] When the electron concentration is low, the above mechanism ceases to be significant; at the point where the gas density is high and the temperature is relatively low, the more significant mechanism of electron attachment to either neutral O_2 or O dominates electron removal.[†] For blunt bodies the chemical relaxation processes in the inviscid region persist over a significant distance behind the body, and hence the interaction of the chemically active inviscid streamlines with the growing inner wake core should be taken into account. These processes are especially significant for large bodies for which boundary-layer effects are negligible.[71,140]

For slender cones the temperature in the inviscid region is rather low, and there are no significant dissociation and ionization phenomena. The assumption of a nonreacting inviscid region is then appropriate.

The importance of the electron attachment mechanism was recognized very early when some first estimates were made at the Lincoln Laboratory,[164][‡] but it is only recently that more sophisticated calculations and correlations have been undertaken. Regardless of their method, all workers[††] agree that the molecular oxygen attachment becomes important below an

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[†]Both O and O_2 are capable of removing electrons with the same efficiency. In an actual reentry, however, O_2 is by far more abundant than O in those regions where electron attachment is important. Along the axis downstream, attachment to atomic oxygen starts to occur at about 100 base diameters, whereas the more vigorous attachment to molecular oxygen starts at about 1000 diameters.

[‡]Since June 1959, the Lincoln Laboratory of MIT has published semiannual reports on its reentry physics program (recent reports are classified). The student of hypersonic wakes will find these reports an excellent source of information, especially on experimental programs both at the laboratory site and at Wallops Island, where experiments on actual reentries have been conducted.

[††]References 93, 121, 126, 129, 140.

altitude† of 150 kft and at axial temperatures of about 800° K and below. When this mechanism takes over, the decay of electron concentration falls drastically. See Fig. 15. As pointed out by Zeiberg,[131] the "outer edges" of the wake are the first to feel the mechanism of oxygen attachment, because the lower temperatures will be attained there first. In this reference, it is found that by using a simplified version of the mass-action law, where electrons attach to molecular oxygen, the electron concentration can be correlated by the velocity decay alone, given an altitude and the conditions at the initiation of the attachment effects.

In their study of the chemistry of the turbulent core behind a sphere, Webb and Hromas[71] consider the charge exchange reaction, $O_2^- + O \rightleftarrows O_2 + O^-$, apart from the attachment reactions already mentioned. It turns out that the results for electron concentration decay are quite sensitive to the inclusion of this reaction. Because the activation energy for this reaction is not known, the calculations were performed with two different values, 0 and 0.5 ev. The first value gives a much faster decay than the second, since the removal of O_2^- is more efficient in the former. The second value yields results in good agreement with the experimental data obtained by Labitt[111] using an "improved"‡ UHF cavity probe. The calculations and experimental data refer to spheres of 3/16 in. diameter flying at a Mach number of 16.8 and at an ambient pressure of 20 mm. On the other hand, Zeiberg and Bleich[136] compare their results, obtained with a finite difference method, with Labitt's data at the same flight conditions but with an aluminum ablating sphere of 0.26 in. diameter. The agreement is rather good for the first 1,500 diameters; above this the experimental data fall below the theoretical

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†Aside from order-of-magnitude arguments that can be deduced from the reaction rate equation,[93,164] Zeiberg,[131] after a detailed computation, shows no attachment effects at an altitude of 180 kft (for a 10 deg cone and 1 ft base radius).

‡In contrast to the VHF cavity probe used by Labitt and Herlin,[135] these last experiments were conducted with aluminum spheres of ¼ in. diameter, flying at velocities between 15 and 16 fps at an ambient density of about 14.5 mm Hg. These data are in good agreement with a rough closed form expression derived by Lees.[93] It is now recognized, however, that the data of Refs. 135 and 111 are not accurate because of large variations from run to run, presumably due to contamination.

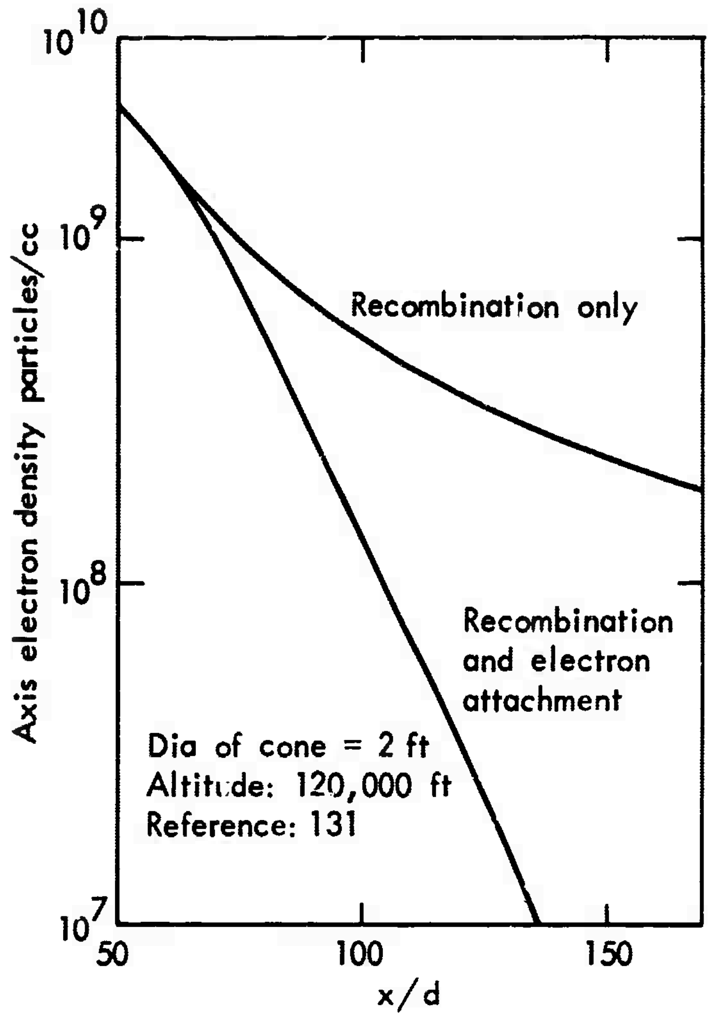

Fig. 15—The effect of oxygen-electron attachment on the electron density along the wake axis behind a 10 deg cone

curve by a factor of ten at 3,000 diameters. It should be noted that the Zeiberg and Bleich theory does not contain the charge exchange reaction.

In Figs. 16 and 17 a comparison is made of theoretical results obtained by Webb and Hromas,[165] based on the scheme presented in Ref. 71, with some unpublished data by Slattery and Kornegay of the Lincoln Laboratory. In view of the uncertainties in our knowledge of the fluid mechanics of the base flow and the neighborhood of the recompression region, to say nothing of the lack of adequate information on the compressible turbulent mechanism, such partial agreements of theory and experiments, as shown in Figs. 16 and 17, although reassuring, should not be taken as proof that the input assumptions and the postulated mechanisms are justified. The same statement is appropriate when comparing theoretical wake velocities with experiments. Such a comparison is shown in Figs. 18 and 19 for cones and spheres, respectively.

Few calculations seem to be available for the optical and infrared radiation in wakes. The earliest is due to Feldman;[4] it deals with laminar, inviscid, and thermodynamic equilibrium conditions. The most recent one is due to Hundley[166] with computations performed in the nonequilibrium regime. Pure air is taken as the working fluid and the following three chemiluminescent reactions are identified as the most important ones: the Lewis-Rayleigh nitrogen afterglow, a reaction producing the blue nitric oxide afterglow, and, finally, the so-called NO_2 continuum reaction. Hundley's calculations are made for the two extreme cases of "inviscid random convection" and "homogeneous mixing" for the turbulent transport attributed to Lin and Hayes.[140] These concepts will be discussed in the following section. To simplify the computations, Hundley assumes that a cylindrical region surrounds the axis over which the axial values for the three important species concentrations (N, N_2, and O) and temperature hold everywhere. Altitudes between 100 kft and 200 kft are considered for blunt bodies of 1 ft radius. His results show a strong dependence on the details of the turbulent mixing process. This is one more case in which more confidence should be established in the fluid mechanical concepts before other phenomena can be understood.

The problem of contamination of pure air has also attracted attention.

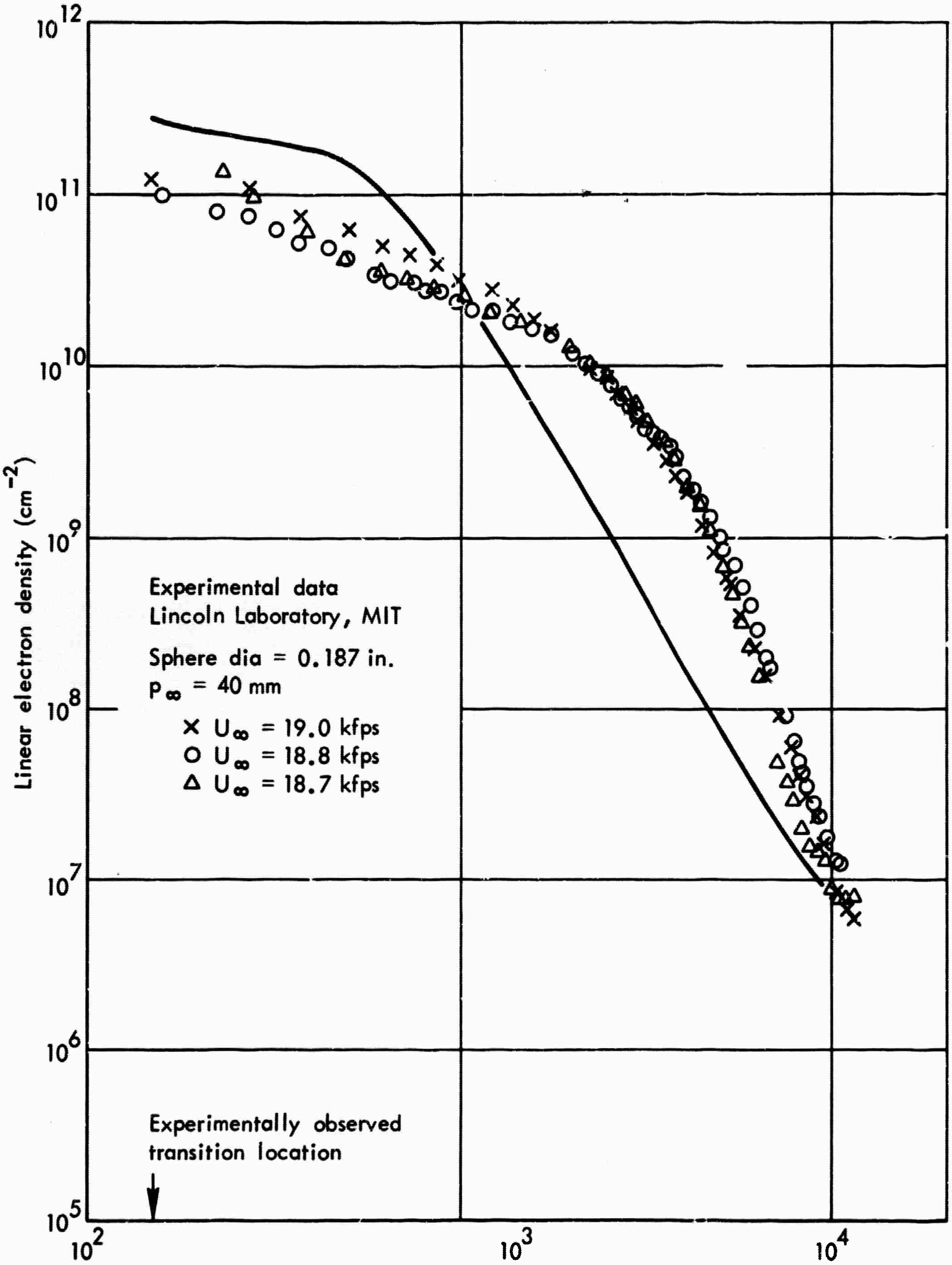

Fig. 16—Comparison of the theory of Webb and Hromas[165] with the experiments of Slattery and Kornegay (free stream pressure 40 mm)

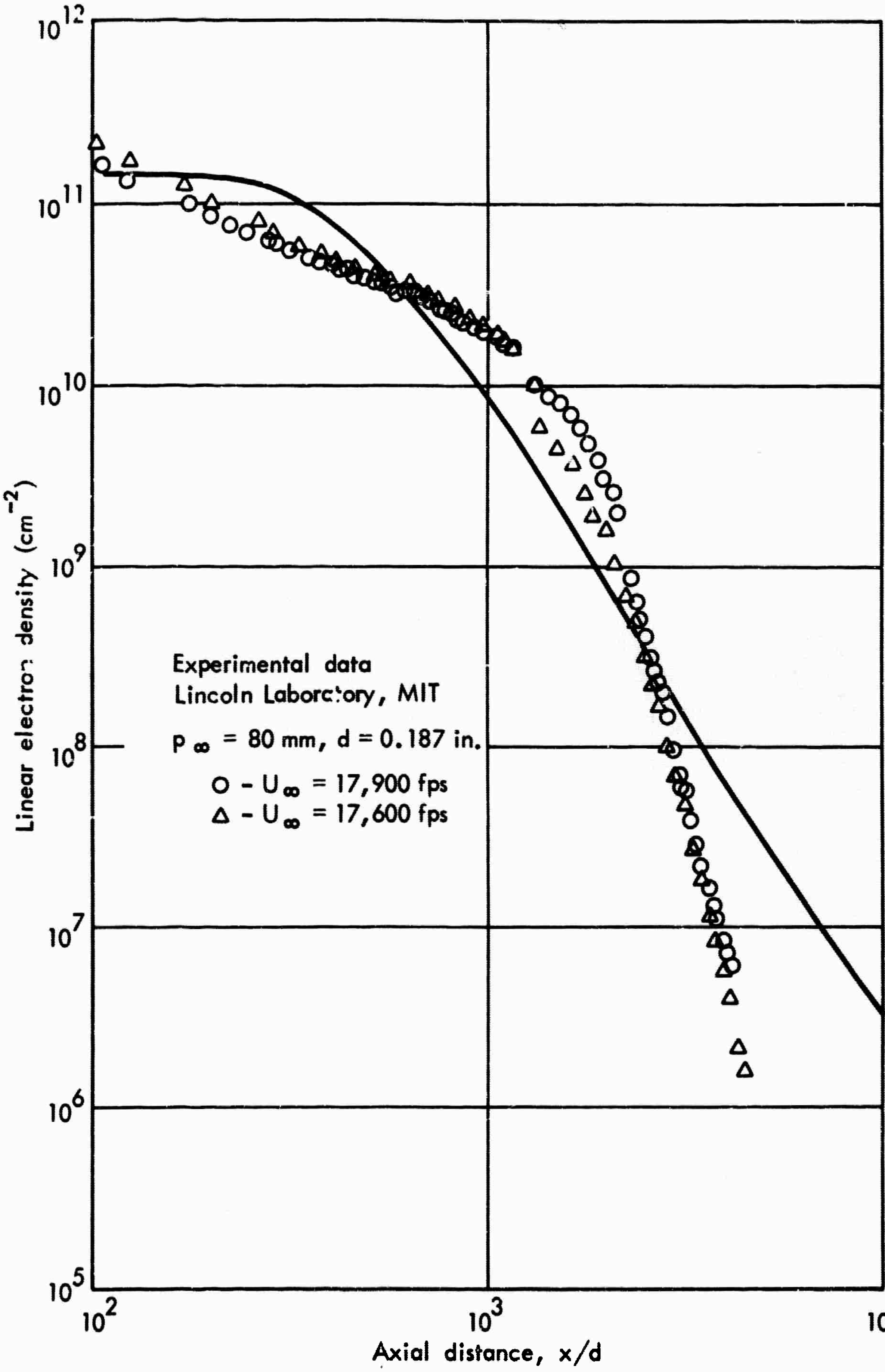

Fig. 17—Comparison of the theory of Webb and Hromas[165] with the experiments of Slattery and Kornegay (free stream pressure 80 mm)

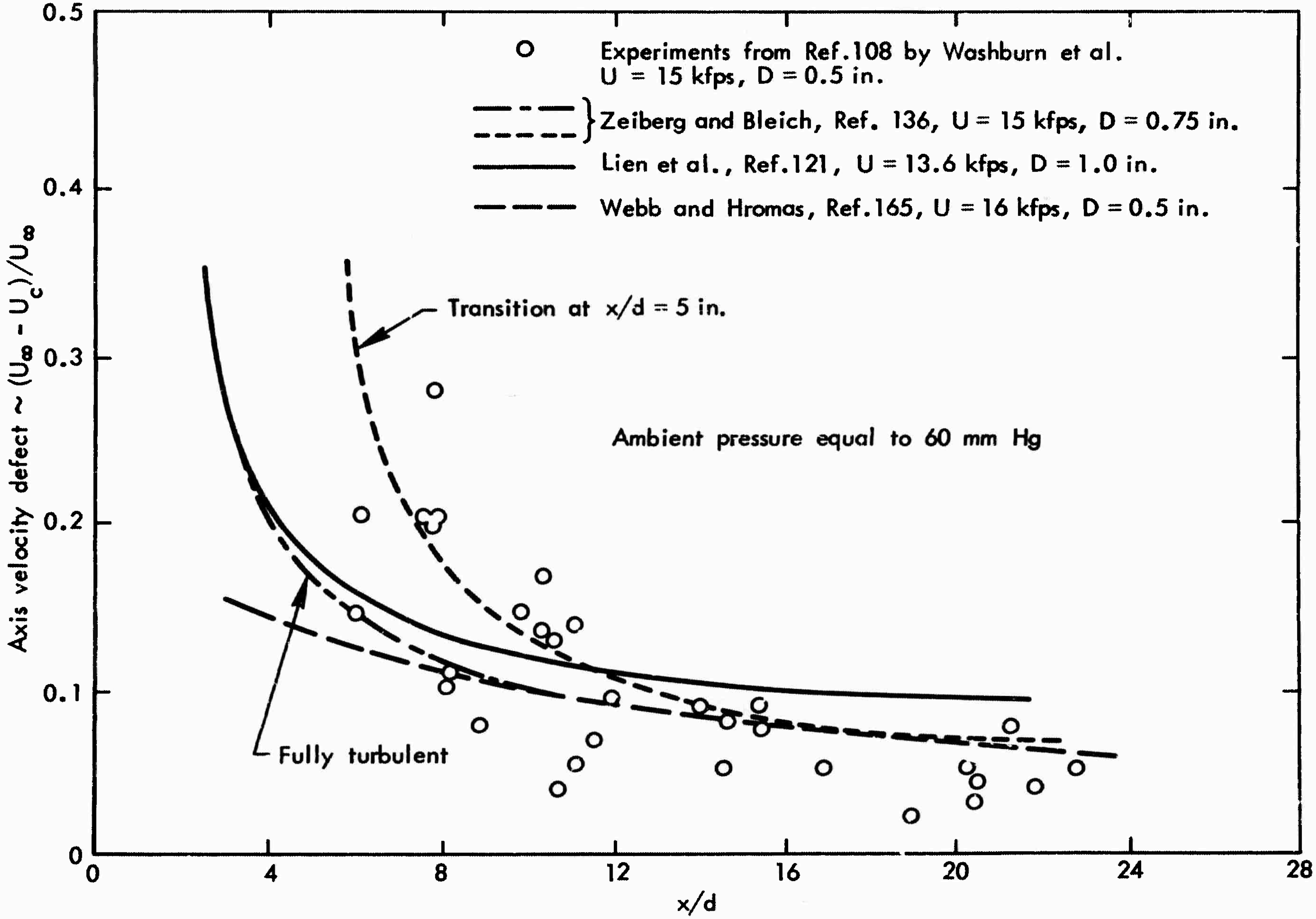

Fig. 18—Comparison of different theories with experiment for the velocity defect in the wake of a 15 deg cone

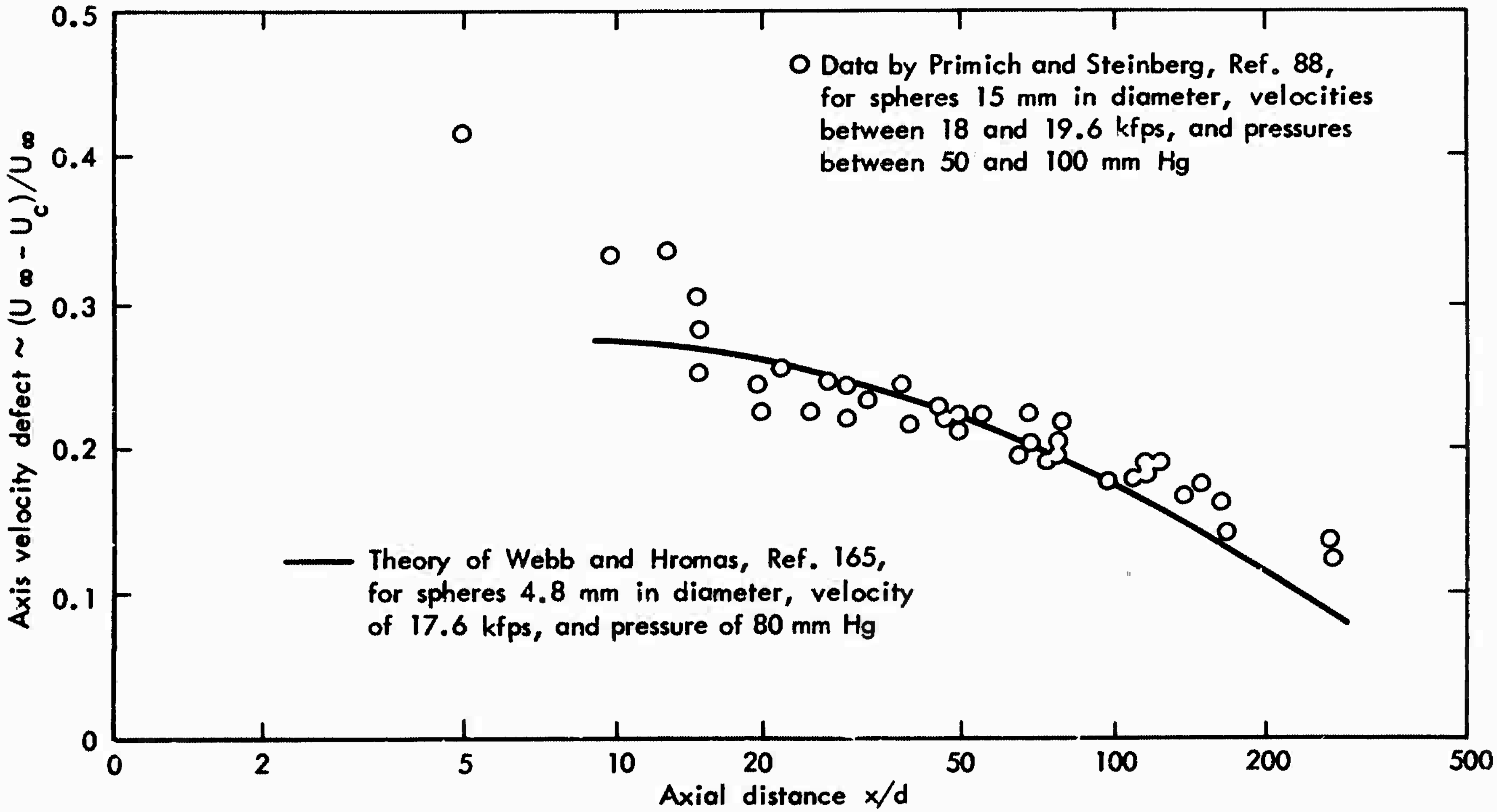

Fig. 19—Comparison between theory and experiment for velocity defect in the wake of a sphere

As early as 1960 Short and Hochstim[119] performed such a study; Li[123] has also undertaken work for slender body laminar and turbulent nonequilibrium wakes when sodium is the main contaminant. As expected, the trails become much longer when compared with wakes of pure air. Sodium contamination has also been treated by Webb and Hromas[71] using an integral technique; slender and blunt bodies are examined.

D. Fluctuations

The interaction of an electromagnetic beam with the electron cloud of a reentering hypersonic object will yield a signal that presumably should be helpful in determining the reentering object's characteristics that are not directly observable, such as its weight. The radar cross section from such an interaction was of interest to meteor physicists long before the fluid mechanics of hypersonic wakes started being studied in a systematic way. For a number of references in the area of meteor wakes, one could consult the work of Lin.[167] From the point of view of radar signatures of hypersonic vehicles, the work of Herlin,[168] Lin et al.,[169] Lin,[167] Menkes,[170] and Weil,[171] among others, could be mentioned. In this section we are only interested in the experiments and calculations performed for the determination of the root mean square of the electron density fluctuations that enter into the calculation of the radar cross section of an underdense trail. From an intuitive point of view, it follows that these fluctuations are closely connected to the temperature fluctuations. Slattery and Clay[79] and Clay et al.,[100] using the fact that schlieren and shadowgraph techniques depend on the gas density variation in the trail, have attempted to recover information concerning the gas density fluctuations from the photographic plates. For this purpose they use densitometer tracings taken along the axis of a turbulent trail as it is shown on a schlieren photograph.† The sensitivity of the measurements is such that only the largest eddies are resolved. From such tracings autocorrelation lengths can be obtained at different stations downstream in the wake. By this technique it is found that for

†It goes without saying that the information obtained from such a technique is an integral taken over the whole width of the trail at any given station.

the same drag coefficient and same flight conditions the autocorrelation function depends strongly on geometry; in fact, cones give higher autocorrelations than spheres. A study of the root mean square of the mass density fluctuations normalized with the mean mass density has also given the result (from a limited number of experimental data, it is true) that for spheres the maximum fluctuation lies at about 400 diameters downstream and has a value of about 0.90, whereas for a 25 deg cone the same value at the maximum is observed at about 50 diameters downstream. Demetriades,[90] working with a hotwire anemometer in a hypersonic wind tunnel, gives fluctuation values lying between 0.10 and 0.40. When the data of Slattery and Clay are lowered[†] by a factor of $\sqrt{2}$, the agreement is not bad, considering the difference in the two techniques.

The above discoveries have stimulated some theoretical work for their justification. Proudian and Feldman[172,173] and Proudian[139] use a "marble cake" structure,[‡] in which a granular gas mixture of the same fluid exists at two different temperatures. They estimate, on the average, that a certain fraction ϵ of the volume of a slice of the wake per unit length is occupied by the cold gas and the remainder by the hot. The hot gas is assumed to have the temperature of the wake center, whereas the cold one has the temperature at the turbulent front. By some simple arithmetical considerations in which a finite "lag time" is introduced for one of the fluids to be homogeneously mixed, it is easy to relate the fraction ϵ, the hot-to-cold temperature ratio, and the mass density root mean square. Some arbitrary constants, however, are needed for the prediction of the root mean square density fluctuations. These constants are chosen to fit the observations. This last step highly detracts from the success of the model.

To answer the same question, Lin and Hayes[140] examine two limiting models. The first one is what they call "inviscid random convection." In this model it is assumed that the eddy motion simply stirs up and redistributes the initial large-scale inhomogeneities across the wake. It is thus assumed that the mean flow properties across the wake are given by

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[†] The discrepancy is due to a numerical mistake. (Private communication to the author by Slattery.)

[‡] This nomenclature is attributed to Herlin (see Ref. 174).

the arithmetical averages of the corresponding flow variables in an equivalent inviscid laminar flow over the same width of the wake; the mean square fluctuations will then be the differences of the various flow properties from the mean, which is computed as stated above. It is claimed that the above model corresponds to the case of random convection discussed by Obukhov[175] and Corrsin.[176] At the other extreme one has the model of "homogeneous mixing," in which it is assumed that new fluid crossing the turbulent boundary is instantaneously and thoroughly mixed with the old fluid already inside the boundary. Thus, a very fast viscous dissipation takes place that dampens all fluctuations. The no-dissipation model should hold during the beginning stages of growth, whereas the dissipative model would hold for the later stages.

Lin[177] has recently proposed a "semi-statistical" theory, which allows an interpolation between the two extreme cases mentioned above. He studies in detail the history of a fluid element during its flight from the moment it crosses the turbulent front until it finds itself at a certain station downstream immersed in the turbulently diffusing core. The central idea of the model is the determination of the portion of the fluid element, $1 - \Psi$, that undergoes distortions due to the violent shear forces during its flight. In order to compute Ψ, Lin assumes that the mean square electron concentration in the undistorted element of the fluid is the same as the one prevailing outside the turbulent core. As pointed out by Proudian,† it is not reasonable to assume that the inviscid portion is independent of its kinematic history during its residence inside the turbulent core and that therefore it is independent of the radial distance from the axis, because the outer portions of the fluid are believed to be dissipated less than the inner ones. In fact, Proudian claims that the value of Ψ, estimated by Lin, is at least two orders of magnitude too small beyond the first one hundred body diameters or so downstream. Furthermore, when the root mean square of the electron density fluctuations calculated according to Lin's model are plotted in the downstream direction, this quantity follows a serpentine course in the direction of the "inviscid

†Private communication.

random" model (which exhibits a monotonic increase). If one allows the assumption that the relative electron density fluctuations are equal to the relative mass density fluctuations, this result contradicts the existing experimental data, the general behavior of which has already been discussed.

A more straightforward point of view, consistent with our understanding of incompressible wake turbulence, has been adopted by Webb.[178] Since the model of self-preservation or local similarity has shown at least some partial success in predicting such gross characteristics as turbulent core growth, the consequences of assuming this model are examined. In this model the velocity fluctuations are directly related to the local velocity deficiency between the axis and turbulent front; hence one can also directly relate the mass density fluctuations to downstream distance. A coefficient of proportionality is needed, and its value is taken to be the same as in the incompressible case, and thus no arbitrary constants are needed for the root mean square fluctuation history of the mass density. Webb's calculations for a sphere show a maximum in this quantity at about 400 diameters downstream (exactly where experiments place it); however, the relative value is found to be only about 0.2, whereas Slattery's experiments yield values near 0.6.

The extent to which the root mean square of the electron density fluctuations can be made equal to the root mean square of the mass density fluctuations has been examined by Demetriades,[137] who used the Saha equation and hence assumed thermodynamic equilibrium. In this particular case he showed that this equality is very likely to exist for the conditions of interest in hypersonic wakes.

V. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

Progress undoubtedly has been made during the last five years during which hypersonic wakes were investigated in a systematic way. If nothing else, one can state that the general problem has been delineated in a number of areas, with the relative importance of each area determined with respect to the others. So, although we do not know the exact answers in the areas pertaining to, say, a separation corner, base, recirculation and recompression regions, turbulence, and chemistry, at least we seem to know in which direction one affects the other.

The fundamental difficulty in the wake problem lies in the fact that the well-established asymptotic techniques that form the backbone of fluid mechanics cannot be applied with confidence. For instance, strictly speaking, boundary-layer theory should not be used in the region between the base and the neck, since the Reynolds number is not uniformly high. Also, inviscid theory cannot be justified in determining the expansion around a corner of separation, nor can it be applied in the recompression region around the neck. Yet all of the analytic attacks available so far have been undertaken with asymptotic methods that are valid either for very high or very small Reynolds numbers, absence of viscous forces, etc., in the hope that in some limited geometry and flow conditions these theoretical results will be justified by experiments (and indeed some of them are).

We now proceed to a quick examination of the different regions to which the hypersonic wake has been delineated. The answer to the problem of the upstream influence of a separation point remains completely unknown. Apart from the qualitative statement that its influence should be smaller as the Reynolds number becomes higher, nothing more specific can be said.

For simplicity, consider the case of a body of length L. Let us define two Reynolds numbers, the first based on the length of the body from leading to trailing edge L (say, Re_L), and the second based on the boundary-layer thickness δ determined at the trailing edge (say, Re_δ). Let us also set the distance upstream from the point of separation over which the existence of the separation is felt equal to λ. In general, for very low Reynolds numbers (Re_L of order one), when the viscous forces are

dominant everywhere in the neighborhood of the body, we shall expect the flow field to be characterized only by the length of the body L (apart from the thickness of the body), and we cannot speak of a second characteristic length λ, since in this case λ merges with L ($\lambda \sim L$). For (Re_L) high enough for the boundary-layer approximation over the body to apply, the distortion around the point of separation will be dictated by the magnitude of the boundary-layer thickness δ. If we find that a Reynolds number based on δ is a number of order one, then we shall expect, as in the case of slow flows, that λ will be of the same order as δ ($\lambda \sim \delta$); if on the other hand we find that $(Re_\delta) \gg 1$, then the flow at the point of separation will be determined locally by a boundary-layer type of expansion, and as a result $\lambda \sim \sqrt{\delta}$. It is thus seen that in an expression like $\lambda/L \sim (Re)_L^{-m}$ it is unlikely that a single value for the power m will give a satisfactory answer for all Reynolds numbers. Figure 20 exhibits the different regimes as outlined above.

The solution to the problem of expansion around a corner of separation is equally unknown. The vast extent to which this expansion is influential in determining the dividing streamline velocity, and hence the conditions at the neck and the resulting development of the diffusing core, has been amply demonstrated by the work of Baum,(31) in which the free shear layer was studied with an initial isentropic expansion of the Blasius profile. It is difficult to believe that such an expansion occurs without the creation of irreversibility. The contrary certainly must be true since the expansion will be dictated largely by the rate at which the shear at the corner will redistribute itself in the free shear layer downstream. In this respect guidance could perhaps be sought from the solution of a problem formulated in terms of a simple viscous turning flow. Unfortunately, even in the case of laminar flow in a curved pipe, the problem is so intractable that no solution is available.†

The free shear layer is intimately coupled with the base, recirculation, and neck regions. So far these regions have been examined independently of each other, followed by attempts to couple them through some

†Neither Miss Dean's(179) laborious analytic attack for the extreme case of small curvatures, nor Adler's(180) boundary-layer type of solution for high curvatures yields the answer. Only an empirical correlation is possible from the available experimental data on skin friction coefficients.

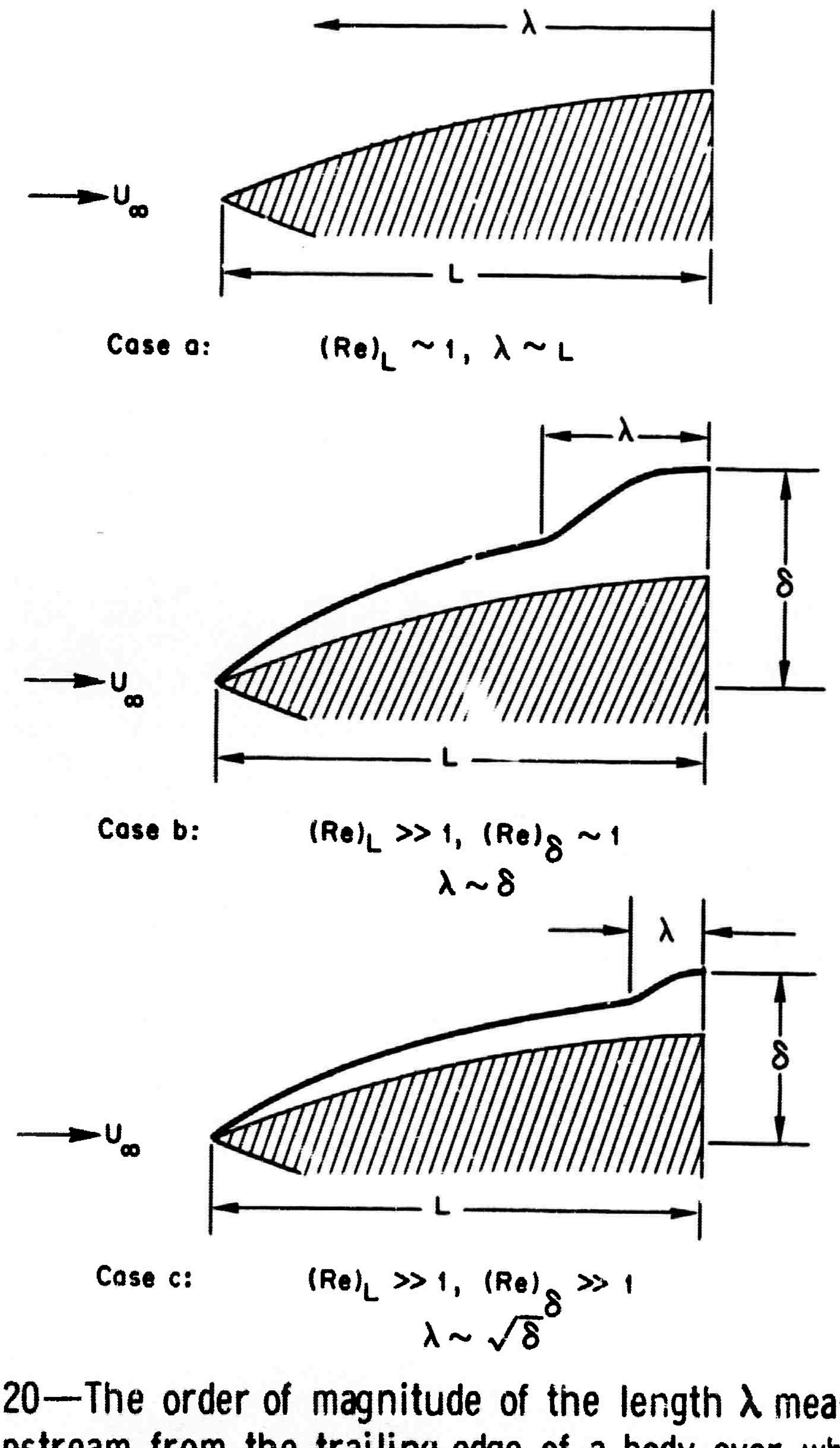

Fig. 20—The order of magnitude of the length λ measured upstream from the trailing edge of a body over which the influence of the trailing edge is observable

kind of a "reasonable" condition. Because most of the available solutions are perturbations about high Reynolds number limits, and because, especially at high altitudes, the Reynolds number enters strongly in such quantities as wake turning angles, base pressure, etc., more solutions should be sought in the Stokes and Oseen's flow regimes. On the other hand, one must keep in mind the difficulties that lie between the limits of Stokes' flow and Prandtl's boundary-layer theory, difficulties well known to fluid mechanists; there is no simple answer for even such well-defined geometries as a sphere when the Reynolds number is neither 1 nor 2, nor 10^5. It is likely that for a large number of geometries and flight conditions the region between the base and the neck of a hypersonic wake will fall somewhere between the Stokes and boundary-layer limits. This possibility and its consequences should be kept in mind when analyzing this region.

Transition to turbulence has been studied in the laboratory for a great variety of geometries and flow conditions. As has already been pointed out, most of these studies are visual observations of the roughness of the edge of the diffusing core and as such are limited to the detection of eddies of large size. Measurements of turbulent correlations are badly needed, difficult as they seem to be. Some theoretical progress has also been made, but surely more work is necessary. From the point of view of the linear small disturbance theory, the results of Gold[97] are illuminating, but limited by the fact that a Gaussian profile for the velocity field and temperature has been assumed. This distribution is not realistic in the neighborhood of the neck, where transition for blunt bodies most likely occurs. Anticipating that accurate, laminarly diffusing core profiles will be established eventually, it would be of interest to develop general approximations for the critical Reynolds numbers for transition to turbulence corresponding to wakelike profiles. Such approximations are already available by Lin[181] on boundary layers for quick estimates of the point where transition occurs.

In general, shear turbulence is, and probably will remain, an unsolved theoretical problem. For the present, an effort is being made, in the case of compressible turbulent wakes, to bring ourselves to the same level of ignorance we seem to command in the incompressible case. Unfortunately,

there are no experiments yet available that can tell us something about the structure of turbulence behind hypersonic turbulent wakes. Such experiments, difficult as they seem to be, are sorely needed, and no number of electronic computations will ever substitute for them; we have a large number of numerical results based on at least five different models of turbulence, but a pause should be taken from the computer until experiments shed some light. All of these models are based on Boussinesq's and Prandtl's empirical concepts. Logically, one is forced to admit that the artificial mechanisms of eddy-diffusivity and mixing length explain nothing of the true nature of turbulence. In essence, they are used to provide definitions and numerical values for quantities whose intrinsic existence can be justified only by the fact that they sometimes provide adequate answers in much wider regions of observation without additional forcing, although they are actually adjusted to satisfy the experimental evidence only in a restricted area. The merit of a semi-empirical theory is judged by the degree of universality of the answer for a minimum number of factors of adjustment.

Judged from this angle, the success of the hypersonic theories (based on the extension of concepts, such as the ones of Refs. 9 and 10, that are valid for subsonic wakes) in predicting growth rates for the turbulently diffusing core has certainly been overrated. It must be kept in mind that comparison with experimental growth data is hardly a sensitive test (even assuming that the theoretically predicted wiggles, such as the ones shown in Fig. 14, are physically there), since the quantity tested is the distribution of momentum in bulk. Furthermore, it would be unwise to assume that because of this success the theory accurately predicts such parameters as temperature, electron density, fluctuations, and the like. In fact, even though it has been possible for four different research groups to report agreement in theoretical wake calculations undertaken independently starting with the same geometry and flow conditions, this success is meaningless when there is no assurance that the initial conditions chosen are correct and that the fluid mechanical and chemical models are realistic.

It is peculiar that the structure of the turbulent intermittency in the hypersonic wakes does not appear to have been studied at all. Yet there are enough photographs and motion pictures from which a great deal

of useful information, such as the size of plumes and characteristic frequencies, could be recovered. Apart from the fact that sphere wakes seem to have a finer structure than cone wakes, and that the latter have much larger plumes (more intermittency[80]), nothing more quantitative seems to exist. It is obvious, however, that the very important mass and electron density fluctuations could perhaps be related to the intermittency phenomena in a way that could aid in the construction of a model that contains at least some apparent and measurable elements of the structure of turbulence, in contrast to the models proposed so far in the literature. These results would be limited to the scale of the roughness visually observed.

Uncertainty in the knowledge of air chemistry as such does not seem to be as high as uncertainty in the fluid mechanical models to which it is coupled. The extent by which the steady-state chemical reaction rates should be modified in fields of fluctuating temperatures still remains an open question. As pointed out by Ferri et al.,[150] as long as the energy content of the fluctuations is small compared to the average static enthalpy, the molecular collision processes might remain unaffected. One can add that a more stringent condition for the validity of the last statement is to ascertain that the energy content of the fluctuations is small compared to the energy levels needed for additional reactions to occur, other than the ones based on the value of the average static enthalpy. In the absence of a definite answer, the steady-state chemical reaction rates are used even in the presence of turbulence. Such an application could perhaps be justified in the colder parts of the wake, for which the chemical reactions are mostly exothermic with a weak dependence on temperature. However, a significant mistake can be made in the hot near wake where an opposite condition prevails. This is particularly serious because of the many other uncertainties of fluid mechanical nature in the first few characteristic lengths behind the body.

Another matter that must be settled is whether all the important reactions relevant to a given "observable" have been taken into account. For instance, we have seen in the work of Webb and Hromas[126] that the influence of the oxygen charge exchange reaction on the electron density

could be very pronounced; yet Ref. 126 is the only work that considers this factor. Whether other reactions are equally important should be considered. In all fairness, at the present time chemistry is mainly in the hands of workers with engineering backgrounds; what is needed is the interest of more physical chemists and physicists.

The following remarks are made in answer to the question of where more work should be done in the area of hypersonic wakes. The region of the upstream influence of a corner of separation needs further study. In the regime lying between the Stokes' flow and Prandtl's boundary-layer limits, boundary conditions should be assumed resembling the ones met in the flow region between the base and the neck. Transition to turbulence of flows with wakelike profiles different from Gaussian should be investigated. Also extensive experimental investigation is needed of the near wake flow in the laminar, transitional, and turbulent regimes of the wake; in this area attention should be focused on obtaining average local properties, such as pressure, mass and electron density, temperature velocity, and the like, along with fluctuations and the most important correlations for the above quantities. Guided by experiments, theoretical models should be constructed for the investigation of the turbulently diffusing wake core. Theoretical and experimental work is also needed for the establishment of the role of the temperature fluctuations in the chemical reaction rates. Finally, enough material has been accumulated in the experimental determination of the growth of turbulent wake cores. There is no need for more work on the order of sizes, general geometries, and flow conditions already available. The same is true for calculations of laminar, turbulent, equilibrium and nonequilibrium far wakes based on uncertain initial conditions and questionable fluid mechanical models (Crocco integral and eddy-diffusivities are examples). Now is the time of the laboratory, and those who have computers should be restrained from computing.

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